

## Warm Up

Translate this sentence into symbolic logic, and describe a weather pattern and transportation method that causes the proposition to be false.

It is snowing today, and if it is raining or snowing then we won't walk to school.

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## Properties of Logical Connectives

These identities hold for all propositions  $p, q, r$

- **Identity**
  - $p \wedge T \equiv p$
  - $p \vee F \equiv p$
- **Domination**
  - $p \vee T \equiv T$
  - $p \wedge F \equiv F$
- **Idempotent**
  - $p \vee p \equiv p$
  - $p \wedge p \equiv p$
- **Commutative**
  - $p \vee q \equiv q \vee p$
  - $p \wedge q \equiv q \wedge p$
- **Associative**
  - $(p \vee q) \vee r \equiv p \vee (q \vee r)$
  - $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
- **Distributive**
  - $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
  - $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
- **Absorption**
  - $p \vee (p \wedge q) \equiv p$
  - $p \wedge (p \vee q) \equiv p$
- **Negation**
  - $p \vee \neg p \equiv T$
  - $p \wedge \neg p \equiv F$

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## Our First Proof

$$(a \wedge b) \vee (\neg a \wedge b) \vee (\neg a \wedge \neg b) \equiv$$

None of the rules look like this

Practice of Proof-Writing:

**Big Picture**...WHY do we think this might be true?

The last two "pieces" came from the vacuous proof lines...maybe the " $\neg a$ "  $\equiv (\neg a \vee b)$  came from there? Maybe that **simplifies** down to  $\neg a$

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## Converse, Contrapositive

**Implication:**

If it's raining, then I have my umbrella.

$$p \rightarrow q$$

**Contrapositive:**

$\neg q \rightarrow \neg p$  If I don't have my umbrella, then it is not raining.

**Converse:**

If I have my umbrella, then it is raining.

$$q \rightarrow p$$

**Inverse:**

$\neg p \rightarrow \neg q$  If it is not raining, then I don't have my umbrella.

**How do these relate to each other?**

$p$	$q$	$p \rightarrow q$	$q \rightarrow p$	$\neg p$	$\neg q$	$\neg p \rightarrow \neg q$	$\neg q \rightarrow \neg p$
T	T						
T	F						
F	T						
F	F						

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