

$p \rightarrow q$ wording

Implication:

p implies q

whenever p is true q must be true

if p then q

q if p

p is sufficient for q

p only if q

q is necessary for p

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Implications are super useful, so there are LOTS of translations.
You'll learn these in detail in section.

A More Complicated Statement

"Robbie knows the Pythagorean Theorem if he is a mathematician and took geometry, and he is a mathematician or did not take geometry."

Is this a proposition?

We'd like to *understand* what this proposition means.

In particular, is it true?

Order of Operations

Just like you were taught PEMDAS

e.g. $3 + 2 \cdot 4 = 11$ not 24.

Logic also has order of operations.

Parentheses

Negation

And

Or, exclusive or

Implication

Biconditional

For this class: each of these is it's own level!

e.g. "and"s have precedence over "or"s

Within a level, apply from left to right.

Other authors place And, Or at the same level – it's good practice to use parentheses even if not required.

Properties of Logical Connectives List

These identities hold for all propositions p, q, r

You don't have to memorize this list!

- **Identity**
 - $p \wedge T \equiv p$
 - $p \vee F \equiv p$
- **Domination**
 - $p \vee T \equiv T$
 - $p \wedge F \equiv F$
- **Idempotent**
 - $p \vee p \equiv p$
 - $p \wedge p \equiv p$
- **Commutative**
 - $p \vee q \equiv q \vee p$
 - $p \wedge q \equiv q \wedge p$
- **Associative**
 - $(p \vee q) \vee r \equiv p \vee (q \vee r)$
 - $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
- **Distributive**
 - $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
 - $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
- **Absorption**
 - $p \vee (p \wedge q) \equiv p$
 - $p \wedge (p \vee q) \equiv p$
- **Negation**
 - $p \vee \neg p \equiv T$
 - $p \wedge \neg p \equiv F$