CSE 311 : Practice Midterm Exam

Name:	**NetID:** @uw.edu

Instructions

- You have eighty minutes to complete this exam.
- You are permitted one piece of 8.5x11 inch paper with handwritten notes (notes are allowed on both sides of the paper).
- You may not use a calculator or any other electronic devices during the exam.
- We will be scanning your exams before grading them. Please write legibly, and avoid writing up to the edge of the paper.
- **Problems are printed on both the front and back of each page!**
- You may also use the last page for extra space, but tell us where to find your answer if it's not right below the problem.
- If you want us to grade something you wrote on scratch paper, put your name and netid on the paper and tell us when you turn in your exam that you have an extra sheet.
- For multiple choice questions, options are shown in ◯ circles; completely fill in the circle for the (one) best answer. If options are shown in ☐ squares, completely fill in the squares for **ALL** correct answers.

Advice

- Remember to properly format English proofs (e.g. introduce all your variables).
- All proofs for this exam must be English proofs, unless we indicate otherwise in a problem.
- If you don't initially know how to approach a proof, it's often helpful to write the start of the proof and put the "target" and conclusion at the bottom.
- Remember to take deep breaths.

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1. Problem 1: Translation

For the rest of the page, let the domain of discourse be mammals. Interpret all sentences below as being in "mathematical English."

You may use the following predicates; the definition for the predicate is given after the colon in the list below.

- `MountainGoat`(x) : x is a mountain goat
- `Sheep`(x) : x is a sheep
- `Climb`(x) : x is climbing
- `SummitsEverest`(x) : x summits Mt. Everest
- `ReachesTop`(x) : x reaches the top
- `Falls`(x) : x falls
- `Knows`(x, y): x knows y
- `Strong`(x) : x is strong

Translate the following English sentences into **PREDICATE LOGIC**.

(a) Every mountain goat that climbs and summits Mt. Everest is strong.

(b) For every mammal that summits Mt. Everest, there is a Mountain Goat that knows it.

Write a **PREDICATE LOGIC** statement equivalent to the English statement below by taking the contrapositive of the implication inside it.

(c) For every mammal, if it falls, then it is a sheep.

Write the negation of the following sentences **IN ENGLISH**. Your English sentences must have negations applied only to individual predicates.

(d) There is a mountain goat that is climbing and reaches the top.

(e) There is a sheep that summits Mt. Everest that every mountain goat knows.

2. Problem 2: Formal Inference Proof

Give a formal symbolic proof of the following statement.

$$\forall x \left[((\forall y\, P(x,y)) \leftrightarrow Q(x)) \to (Q(x) \to \exists y\, P(x,y))\right]$$

You may use the following extra law, in addition to the ones on the reference sheets:
Definition of a biconditional: $p \leftrightarrow q \equiv p \to q \wedge q \to p$

3. Problem 3: Number Theory Proof

Show for all integers a, b, n where $n > 0$ that if $n|(b-a)$ then $n|[(b-2n)-(a+3n)]$.

You must give an **English** proof of the claim.
In this problem, you **may** use the definitions of modular equivalence and divides, and algebra. You **may not** use

other theorems about modular arithmetic unless you re-prove them.

4. Problem 4: Short Answer

(a) Which of the following describes what it means when we write $\exists x \forall y P(x, y)$?

◯ All the values (x) in our domain have the same value y that makes $P(x, y)$ true. (y cannot depend on x)
◯ All the values (y) in our domain have a value x that makes $P(x, y)$ true. (x can depend on y).
◯ There is a value (x) in our domain so that for all values y, $P(x, y)$ is true.

(b) Which of the following expressions are equivalent to $(p \vee q) \wedge v$? Mark ALL that apply.

☐ $(\neg p \to q) \wedge v$
☐ $(q \to \neg p) \wedge v$
☐ $(p + q) \cdot v$
☐ $(p \to q) \wedge v$
☐ $p \vee (q \wedge v)$

(c) Which of the following expressions are **NOT** equivalent to $(p \vee q) \vee r$? Mark ALL that apply.

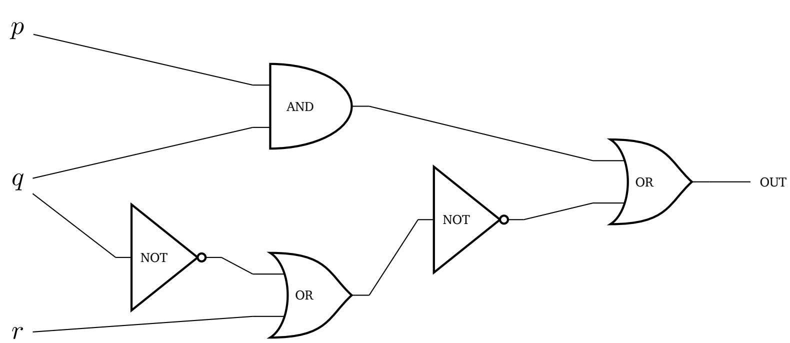
☐ $r \to \neg(p \wedge q)$
☐ $p \vee (q \vee r)$
☐ $(\neg p \to q) \vee r$
☐ $p \vee (\neg q \to r)$
☐ $\neg(p \vee q) \to r$

(d) Which of the following is the **DNF** of the given truth table?

p	q	r	Output
T	T	T	T
T	T	F	F
T	F	T	F
T	F	F	F
F	T	T	T
F	T	F	F
F	F	T	T
F	F	F	F

◯ $(p \vee q \vee r) \wedge (\neg p \vee q \vee r) \wedge (\neg p \vee \neg q \vee r)$
◯ $(p \wedge q \wedge r) \vee (\neg p \wedge q \wedge r) \vee (\neg p \wedge \neg q \wedge r)$
◯ $(\neg p \wedge \neg q \wedge \neg r) \vee (p \wedge \neg q \wedge \neg r) \vee (p \wedge q \wedge \neg r)$
◯ $(\neg p \vee \neg q \vee \neg r) \wedge (p \vee \neg q \vee \neg r) \wedge (p \vee q \vee \neg r)$

(e) Which of the following is the circuit below equivalent to?



◯ $(p \wedge \neg q) \vee \neg(\neg q \vee r)$
◯ $(p \wedge q) \vee (\neg q \vee r)$
◯ $(p \wedge q) \vee \neg(\neg q \vee r)$
◯ $(p \wedge q) \wedge \neg(\neg q \vee r)$

(f) You wish to show "For every integer, if it is divisible by 10 then it is even" with a proof by contrapositive. State the contrapositive of the claim in English.

(g) A reliable source tells you the following statement is true: "If a student is taking 311, then they know DeMorgan's Law." What can you conclude about the statement "If a student knows DeMorgan's Law, then they are taking 311." ?

◯ The second statement must be true.
◯ The second statement cannot be true.
◯ The second statement might or might not be true.