CSE 311 : Winter 2024 Midterm Exam

Name:	**NetID:** @uw.edu

Instructions

- You have ninety minutes to complete this exam.
- You are permitted one piece of 8.5x11 inch paper with handwritten notes (notes are allowed on both sides of the paper).
- You may not use a calculator or any other electronic devices during the exam.
- We will be scanning your exams before grading them. Please write legibly, and avoid writing up to the edge of the paper.
- **Problems are printed on both the front and back of each page!**
- You may also use the last page for extra space, but tell us where to find your answer if it's not right below the problem.
- If you want us to grade something you wrote on scratch paper, put your name and netid on the paper and tell us when you turn in your exam that you have an extra sheet.
- For multiple choice questions
  - If options are shown in ◯ circles, completely fill in the circle for the (one) best answer.
  - If options are shown in ☐ squares, completely fill in the squares for **ALL** correct answers (there may be more than one).

Advice

- Remember to properly format English proofs (e.g. introduce all your variables).
- All proofs for this exam must be English proofs.
- **We give partial credit for the beginning and end of a proof**. Even if you don't know how the middle goes, you can write the start of the proof and put the "target" and conclusion at the bottom.
- Remember to take deep breaths.

Question	Max points
Rubik's Cubes	18
A Proof	16
Induction	20
Short Answer	16
Total	**70**

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1. Rubik's Cubes!! [18 points]

For the rest of this problem, let the domain of discourse be people. Interpret all sentences below as being in "mathematical English."

You may use the following predicates; the definition for the predicate is given after the colon in the list below.

- $\texttt{Cuber}(x)$: x is a cuber (a cuber is someone who likes Rubik's cubes)
- $\texttt{CanSolve}(x)$: x can solve a Rubik's cube
- $\texttt{AttendsCompetition}(x)$: x attends a competition
- $\texttt{CanSolveBlindfoled}(x)$: x can solve a Rubik's cube blindfolded (you may abbreviate this predicate as CSB)
- $\texttt{Cooler}(x, y)$: x is cooler (more impressive) than y

Use $x = y$ to state that x and y are the same person, and $x \neq y$ to state they are different.

Note: Do not give more information than needed. The correct translation of "Everyone can solve a rubik's cube blindfolded" is $\forall x(\texttt{CanSolveBlindFolded(x)})$. Do NOT add the predicate $\texttt{CanSolve}(x)$ in that answer.

Translate the following English sentences in part (a) and (b) into **predicate logic**.

(a) Every cuber that attends a competition must know how to solve a Rubik's cube.

(b) There are exactly two cubers that can solve a Rubik's cube blindfolded.

Translate the predicate logic statements in parts (c) and (d) into **English**. Your English translation must take advantage of domain restriction where possible.

(c) $\exists x(\texttt{Cuber}(x) \land \neg\,\texttt{CanSolve}(x) \land \forall y((\texttt{Cuber}(y) \land \texttt{CanSolve}(y)) \to \texttt{Cooler}(y, x)))$

(d) $\forall x \exists y(\texttt{Cuber}(x) \to [\texttt{CanSolve}(y) \land \texttt{Cooler}(y, x)])$

Recall the list of predicates from the last page

- $\texttt{Cuber}(x)$: x is a cuber (a cuber is someone who likes Rubik's cubes)
- $\texttt{CanSolve}(x)$: x can solve a Rubik's cube
- $\texttt{AttendsCompetition}(x)$: x attends a competition
- $\texttt{CanSolveBlindfoled}(x)$: x can solve a Rubik's cube blindfolded (you may abbreviate this predicate as CSB)
- $\texttt{Cooler}(x, y)$: x is cooler (more impressive) than y

Use $x = y$ to state that x and y are the same person, and $x \neq y$ to state they are different.

Parts (e) and (f) refer to this predicate statement:

$$\forall x((\texttt{CanSolve}(x) \vee \texttt{AttendsCompetition}(x)) \rightarrow \texttt{Cuber}(x))$$

(e) State the contrapositive of the statement above in **English**. Your solution must explicitly state all quantifiers, have negations applied to individual predicates, and take advantage of domain restriction.

(f) State the negation of the statement above in **English**. Your solution must explicitly state all quantifiers, have negations applied to individual predicates, and take advantage of domain restriction.

2. Number Theory! [16 points]

(a) For all integers p, n, and q where $n > 0$, prove:

$$\text{If } p \equiv 1 \pmod{3n} \text{ and } q \equiv 2 \pmod{4n}\text{, then } 4p + 3q \equiv 10 \pmod{12n}.$$

Hint Use your definitions! [13 points]

(b) Which of the following is true about divides for all integers a, b, c? [3 points]

◯ If $a|b$, then $b|a$.
◯ If $a|b$, then $b \nmid a$.
◯ If $a|b$ and $b|c$, then $a|c$.
◯ If $a|b$ and $b|c$, then $a \nmid c$.

3. Induction! [20 points]

Prove by induction that for all integers $n \geq 1$:

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}$$

This equation can be written equivalently in summation notation as

$$\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}$$

You may use either (or both) formulations in your proof.
Make sure to use the template covered in class, including defining a predicate $P()$.

Use this page to continue your induction proof.

4. Short Answer [16 points]

(a) You wish to show "For every integer, if it is divisible by 10 then it is even" with a proof by contrapositive. State the contrapositive of the claim in English. [4 points]

(b) Which of the following describes what it means when we write $\exists x \forall y P(x, y)$? [2 points]

◯ All the values (x) in our domain have the same value y that makes $P(x, y)$ true. (y cannot depend on x)

◯ All the values (y) in our domain have a value x that makes $P(x, y)$ true. (x can depend on y).

◯ There is a value (x) in our domain so that for all values y, $P(x, y)$ is true.

(c) Which of the following expressions are **NOT** equivalent to $(p \vee q) \vee r$? Mark ALL that apply [2 points]

☐ $r \rightarrow \neg(p \wedge q)$

☐ $p \vee (q \vee r)$

☐ $(\neg p \rightarrow q) \vee r$

☐ $p \vee (\neg q \rightarrow r)$

☐ $\neg(p \vee q) \rightarrow r$

(d) A reliable source tells you the following statement is true: "If a student is taking 311, then they know DeMorgan's Law." What can you conclude about the statement "If a student knows DeMorgan's Law, then they are taking 311." ? [2 points]

◯ The second statement must be true.

◯ The second statement cannot be true.

◯ The second statement might or might not be true.

(e) Which of the following expressions are equivalent to $(p \vee q) \wedge v$? Mark ALL that apply? [2 points]

☐ $(\neg p \rightarrow q) \wedge v$

☐ $(q \rightarrow \neg p) \wedge v$

☐ $(p \rightarrow q) \wedge v$

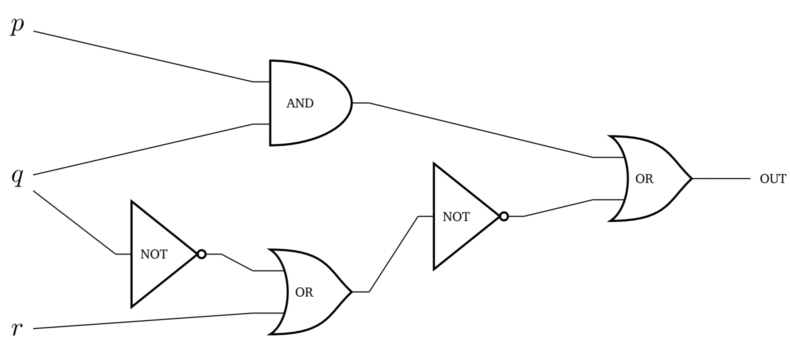
☐ $p \vee (q \wedge v)$

(f) Which of the following is the **DNF** of the following truth table? [2 points]

p	q	r	$(p \to q) \wedge r$
T	T	T	T
T	T	F	F
T	F	T	F
T	F	F	F
F	T	T	T
F	T	F	F
F	F	T	T
F	F	F	F

◯ $(p \vee q \vee r) \wedge (\neg p \vee q \vee r) \wedge (\neg p \vee \neg q \vee r)$
◯ $(p \wedge q \wedge r) \vee (\neg p \wedge q \wedge r) \vee (\neg p \wedge \neg q \wedge r)$
◯ $(\neg p \wedge \neg q \wedge \neg r) \vee (p \wedge \neg q \wedge \neg r) \vee (p \wedge q \wedge \neg r)$
◯ $(\neg p \vee \neg q \vee \neg r) \wedge (p \vee \neg q \vee \neg r) \wedge (p \vee q \vee \neg r)$

(g) Which of the following is the circuit below equivalent to? [2 points]



◯ $(p \wedge \neg q) \vee \neg(\neg q \vee r)$
◯ $(p \wedge q) \vee (\neg q \vee r)$
◯ $(p \wedge q) \vee \neg(\neg q \vee r)$
◯ $(p \wedge q) \wedge \neg(\neg q \vee r)$

Use this page for extra space if you need it. ***And be sure to tell us to look here on the original problem!***