CSE 311 : Autumn 2023 Midterm Exam

Name:	**NetID:** @uw.edu

Instructions

- You have ninety minutes to complete this exam.
- You are permitted one piece of 8.5x11 inch paper with handwritten notes (notes are allowed on both sides of the paper).
- You may not use a calculator or any other electronic devices during the exam.
- We will be scanning your exams before grading them. Please write legibly, and avoid writing up to the edge of the paper.
- **Problems are printed on both the front and back of each page!**
- You may also use the last page for extra space, but tell us where to find your answer if it's not right below the problem.
- If you want us to grade something you wrote on scratch paper, put your name and netid on the paper and tell us when you turn in your exam that you have an extra sheet.
- For multiple choice questions
  - If options are shown in ◯ circles, completely fill in the circle for the (one) best answer.
  - If options are shown in ☐ squares, completely fill in the squares for **ALL** correct answers (there may be more than one).

Advice

- Remember to properly format English proofs (e.g. introduce all your variables).
- **All proofs for this exam must be English proofs.**
- **We give partial credit for the beginning and end of a proof**. Even if you don't know how the middle goes, you can write the start of the proof and put the "target" and conclusion at the bottom.
- Remember to take deep breaths.

Question	**Max points**
Sheep and Mountain Goats	15
A Proof	13
Another Proof	10
Induction	20
Short Answer	12
Total	**70**

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1. Sheep and Mountain Goats [15 points]

For the rest of the page, let the domain of discourse be mammals. Interpret all sentences below as being in "mathematical English."

You may use the following predicates; the definition for the predicate is given after the colon in the list below.

- `MountainGoat`(x) : x is a mountain goat
- `Sheep`(x) : x is a sheep
- `Climb`(x) : x is climbing (Treat "climbs" and "is climbing" as equivalent ideas in this problem)
- `SummitsEverest`(x) : x summits Mt. Everest
- `ReachesTop`(x) : x reaches the top
- `Falls`(x) : x falls
- `Knows`(x, y): x knows y
- `Strong`(x) : x is strong

Translate the following English sentences into **PREDICATE LOGIC**.

(a) Every mountain goat that climbs and summits Mt. Everest is strong.

(b) For every mammal that summits Mt. Everest, there is a Mountain Goat that knows it.

Write a **PREDICATE LOGIC** statement equivalent to the English statement below by taking the contrapositive of the implication inside it. Be sure to write an equivalent statement to the whole sentence, not just the implication inside.

(c) For every mammal, if it falls, then it is a sheep.

Write the negation of the following sentences **IN ENGLISH**. Your English sentences must have negations applied only to individual predicates, but you do not need to use domain restriction.

(d) There is a mountain goat that is climbing and reaches the top.

(e) There is a sheep that summits Mt. Everest that every mountain goat knows.

2. A Proof! [13 points]

Let A, B be arbitrary sets. We have the following claim:

$$(A \cap \overline{B}) \cup (\overline{A} \cap B) \subseteq \overline{(A \cap B)}$$

(a) Translate the claim into predicate logic. [3 points]

(b) Write an **English** proof for this claim. Do not write a logical equivalences proof, or part of a logical equivalences proof—your proof must be in English, though you can still use symbols appropriately. [10 points]

Hint: Parts of this proof are very similar to the set proofs you did on the homework!

Hint: If you get stuck on this proof (or another one), it's ok to skip a problem for a bit and come back. It's also a good idea to write the start and end of the proof at least, even if you don't know what happens in the middle.

3. Another Proof! [10 points]

Show for all integers a, b, n where $n > 0$ that if $a \equiv b \pmod{n}$ then $a + 3n \equiv b - 2n \pmod{n}$.

In this problem, you **may** use

- The definition of equivalence (mod n) (i.e., that $x \equiv y \pmod{n}$ if and only if $n|(y - x)$.
- The definition of divides (i.e., that $x|y$ if and only if there is an integer z such that $xz = y$.)
- The theorem that if $x \equiv y \pmod{n}$ then $y \equiv x \pmod{n}$
- Algebra

You **may not** use other theorems proven or provided in class materials (for example you **may not** use that "if $a \equiv b \pmod{n}$ then $a + c \equiv b + c \pmod{n}$", or "if $a \equiv b \pmod{n}$ then $a \equiv b + n \pmod{n}$"), unless you reprove them.

4. Induction! [20 points]

Consider the following code snippet.

```
int Mystery(int n) {
    if n == 0:
        return 5
    if n == 1:
        return 16
    return 7 * Mystery(n - 1) - 10 * Mystery(n - 2)
}
```

In this problem, we will use `Mystery(n)` to refer to the value returned by the code snippet above when run on input `n`. For example, `Mystery(2)` $= (7 \cdot 16) - (10 \cdot 5) = 62$.

Use induction to show that `Mystery(n)` $= 3 \cdot 2^n + 2 \cdot 5^n$ for all integers $n \geq 0$.

Use this page to continue your induction proof. Here's the code again in case you need to reference it.

```
int Mystery(int n) {
    if n == 0:
        return 5
    if n == 1:
        return 16
    return 7 * Mystery(n - 1) - 10 * Mystery(n - 2)
}
```

Continue your proof below

5. Short Answer [12 points]

(a) You wish to show "For every integer, if it is divisible by 4 then it is even" with a proof by contradiction. Complete **just the first sentence** of the proof (be sure to assume everything you can, and to explicitly include any quantifiers) [4 points]

Suppose, for the sake of contradiction, that...

(b) Which of the following is a multiplicative inverse of $3 \pmod 5$ [2 points]
Hint: You don't have to use the process to find a multiplicative inverse if you remember what a multiplicative inverse is!

◯ 1
◯ 2
◯ 3
◯ 4

(c) Which of the following expressions are equivalent to $(p \vee q) \wedge v$. Mark ALL that apply [2 points]

☐ $(\neg p \rightarrow q) \wedge v$
☐ $(q \rightarrow \neg p) \wedge v$
☐ $(p \rightarrow q) \wedge v$
☐ $p \vee (q \wedge v)$

(d) A reliable source tells you the following statement is true: "If a dog likes purple and gold then it's a husky." What can you conclude about the statement "If a dog is a husky, then it likes purple and gold." ? [2 points]

◯ The second statement must be true.
◯ The second statement might or might not be true.
◯ The second statement cannot be true.

(e) Which of these is the Extended Euclidean algorithm good for. Mark ALL that apply [2 points]

☐ Finding the gcd of two integers
☐ Finding multiplicative inverse of an integer (mod n)
☐ Finding $a\%b$ (i.e., the remainder when you divide a by b)
☐ Determining how many prime numbers exist between two given numbers.

Use this page for extra space if you need it. ***And be sure to tell us to look here on the original problem!***