

CSE 311 : Autumn 2023 Final Exam, Form A

Name:

NetID: @uw.edu

Instructions

- You have one-hour and fifty-minutes to complete this exam.
- You are permitted one piece of 8.5x11 inch paper with handwritten notes (notes are allowed on both sides of the paper).
- You may not use a calculator or any other electronic devices during the exam.
- We will be scanning your exams before grading them. Please write legibly, and avoid writing up to the edge of the paper.
- **Problems are printed on both the front and back of each page!**
- You may also use the last page for extra space, but tell us where to find your answer if it's not right below the problem.
- If you want us to grade something you wrote on scratch paper, put your name and netid on the paper and tell us when you turn in your exam that you have an extra sheet.

Advice

- Remember to properly format English proofs (e.g. introduce all your variables).
- All proofs for this exam must be English proofs.
- We give partial credit for the beginning and end of a proof. Even if you don't know how the middle goes, you can write the start of the proof and put the "target" and conclusion at the bottom.
- Remember to take deep breaths.

Question	Max points
Training Wheels	14
First Proof	11
Models of Computation	13
Induction I	20
Short Answer	11
Induction II	20
Wait, That's Illegal	15
Grading Morale	1
Total	105

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1. Training Wheels [14 points]

Let your domain of discourse be integers. Define the following predicates

- $\text{Divides}(x, y)$ returns true if and only if $x|y$.
- $\text{Even}(x)$ returns true if and only if x is even.
- $\text{Prime}(x)$ returns true if and only if x is prime.
- $\text{IsTwo}(x)$ returns true if and only if x is the number 2.
- You may also use standard arithmetic connectors like $=, <, \geq$, etc.

- (a) Translate this sentence into predicate logic notation. “Strictly between a and b ” means neither a nor b count. [3 points]

For every integer k , there is at least one prime integer j strictly between k and $3k$.

- (b) Take the contrapositive of the following statement (leave your answer as symbols; negations must be applied to single predicates, but you do not need to reformat your answer to show domain restriction). [3 points]

$$\forall y(\text{IsTwo}(y) \rightarrow [\text{Even}(y) \wedge \text{Prime}(y)])$$

- (c) Translate this sentence into predicate logic notation. [3 points]

If x is even and x is prime then x is less than 3 and greater than 1.

- (d) Negate the statement in part (c) (not your answer). Give your answer in English. You must explicitly state all quantifiers (even if you might omit them in everyday English). Additionally “not”s must apply only to single predicates [3 points]

- (e) The (original) statement in (b) is [1 point]

- True
 False

- (f) The contrapositive of the original statement in (b) has: [1 point]

- The same truth value as the original statement in part (b).
 The negation of the truth value of the original statement in part (b).
 It depends on what y is.

2. First Proof [11 points]

Let $A = \{n \in \mathbb{Z} : n \equiv 0 \pmod{30}\}$

Let $B = \{n \in \mathbb{Z} : 5 \mid n\}$

Let $C = \{n \in \mathbb{Z} : 3 \mid n\}$

In the notation above, ‘:’ is “such that” and ‘|’ is “divides.”

- (a) Prove that $A \subseteq B \cap C$ [7 points]

- (b) Does the subset relationship hold in the other direction? That is, is this statement true?:

$$B \cap C \subseteq A$$

The statement is: [2 points]

- True
 False

- (c) Let D, E be sets. If D has eight elements, $D \cup E$ has nine elements, and $D \setminus E$ has four elements, then E must contain four elements. *Hint: It may helpful to visualize this by drawing a Venn Diagram*

The statement is: [2 points]

- True
 False

3. Models of Computation [13 points]

Let L_1 be the language containing all strings over $\{0, 1\}$ such that it satisfies **both** of these two requirements:

- 0 cannot be immediately followed by a 1.
- The sum of all the digits in the string is even.

For example:

- 101 is not in L_1 (the 0 is followed by a 1)
- 00 is in L_1 .
- ε is in L_1 (the sum is 0, which is even).

(a) Write a CFG that generates L_1 . Be sure to tell us which symbol is the start symbol; also include a sentence or two of explanation of how your CFG works. [4 points]

(b) A specific computer only accepts passwords that satisfy the following requirements:

- The password can only use the digits 1, 2, 3
- The password must be at least 4 digits long.
- The password when interpreted as a base-10 number is even.

Write a regular expression for all valid passwords. [3 points]

(c) Let L_2 be the language containing all strings over $\{j, k, m\}$ such that **either**

- the number of j 's is even, **or**
- contains the subsequence km , i.e. there's a k immediately followed by an m .

but not both.

Draw a DFA that accepts L_2 . Include a brief description of how your machine works (You'll probably want about 2-3 sentences total, or a few words for each state about what it does). [6 points]

4. Induction I [20 points]

Consider the following recursive function:

$$f(n) = \begin{cases} 4 & n = 2 \\ 12 & n = 3 \\ \frac{f(n-1) \cdot n}{2(n-2)} + \frac{f(n-2) \cdot n(n-1)}{2(n-3)(n-2)} & n \geq 4 \end{cases}$$

Prove $f(n) = 2n(n-1)$ for all integers $n \geq 2$.

You **must** use induction; be sure to define a predicate $P()$ and use good style.

Hint: Don't distribute (i.e., FOIL) terms before you have to in your inductive step. Look for things that might cancel.

5. Short Answer [11 points]

- (a) Write the first sentence of a proof by contradiction trying to prove “If a graph is bipartite, then it has no odd cycles.” Make any quantifiers in your sentence explicit and be sure you’re following good style for the first sentence of a proof by contradiction. (You do not need to know what ‘bipartite’ means to do this problem.) [3 points]

- (b) Let L_1 be the language accepted by a DFA, and let L_2 be the language accepted by an NFA. What can we say about $L_1 \cap L_2$? [2 points]
- There is a DFA that accepts exactly $L_1 \cap L_2$.
 - There might or might not be a DFA that accepts exactly $L_1 \cap L_2$.
 - There cannot be a DFA that accepts exactly $L_1 \cap L_2$.

- (c) Which of the following describes what it means when we write $\forall x \exists y P(x, y)$ [2 points]
- All the values (x) in our domain have the same value y that makes $P(x, y)$ true. (y cannot depend on x)
 - All the values (x) in our domain have a value y that makes $P(x, y)$ true. (y can depend on x).
 - There is a value (x) in our domain so that for all values y , $P(x, y)$ is true.

6. Fruits and Veggies (Induction II) [20 points]

Consider binary trees that hold fruits and veggies which we'll call produceTrees.

Basis Step: null is a produceTree

Recursive Step 1: If L, R are produceTrees then (L, fruit, R) is also a produceTree

Recursive Step 2: If L is a produceTree then $(L, \text{veggie}, \text{null})$ is also a produceTree.

Intuitively, Recursive Step 1 creates a fruit node with two children and Recursive Step 2 creates a veggie node with only one child.

Using **structural** induction, show that for all produceTrees, if they have f fruits and v veggies then they have $f + v + 1$ copies of null.

Additional space for Problem 6.

7. Wait, that's illegal [15 points]

Choose to do exactly one of these two problems.

- (a) Let $\Sigma = \{0, 1, \#\}$. I.e., our alphabet contains 0 and 1 and a special character #. Let L be the following language:

$$L = \{x : x = 0^k \# 1^{2k}\}$$

That is, each string in L has a single # separating two substrings. The substring before the # is all 0's, the substring after the # is twice as many 1's.

- $00\#1111 \in L$
- $00\#111 \notin L$
- $0\#11 \in L$
- $\# \in L$
- $00\#11\#11 \notin L$

Prove that L is not regular.

- (b) Let $\Sigma = \{3, 4, 5\}$. Define X to be the set of all functions with domain Σ^* and co-domain $\{0, 1, 2\}$. For example, the function "given a string, return its length % 2" is an element of X

Prove, by recreating a diagonalization argument like that shown in class, that $\mathcal{P}(X)$ is uncountable.

I am doing

- part (a)
 part (b)

Additional space for Problem 7.

8. Grading Morale [1 point]

Put something on this page. Depending on your mood, this might be a poem, the statement of your favorite theorem, the statement of your least favorite theorem, or a piece of art. You can also put the name of a staff member that you appreciate. :)

As long as you make some mark on this page, you will get the point.

Looking at these helps keep the TAs happy while grading.

Additional space for any prior problem. Be sure to tell us to look here on the original problem!