Homework 3: Predicate Logic

Special Note: This homework is divided into 2 parts, in an effort to give you practice with and feedback on all the Midterm 1 topics before the exam.
Part 1 (problems 1-6) will be due Wednesday October 15th at 11:59 PM. You will be ready to do everything in part 1 when it is released. Late days for Part 1 will work like they normally do (allowing you to submit until Saturday October 18th).
Part 2 (problems 7-9) covers the last topics that are fair game for Midterm 1. You will know everything you need to do Part 2 after lecture on Monday October 13th. Part 2 will be due Saturday October 18th. To make sure we can give you feedback in time to be useful for midterm studying, **you cannot use late days on Part 2**.

Part 1 Due date: Wednesday October 15th at 11:59 PM
Part 2 Due date: Saturday October 18th at 11:59 PM
Late Submission deadline (both parts): Saturday October 18th at 11:59 PM

If you work with others (and you should!), remember to follow the collaboration policy outlined in the syllabus.
In general, you are graded on both the clarity and accuracy of your work. Your solution should be clear enough that someone in the class who had not seen the problem before would understand it.
We sometimes describe approximately how long our explanations are. These are intended to help you understand approximately how much detail we are expecting. You are allowed to have longer explanations, but explanations significantly longer than necessary may receive deductions.

Be sure to read the grading guidelines on the assignments page for more information on what we're looking for.

1. Coffee Shop Conundrum

The baristas are deciding what kinds of espresso drinks to make after the morning rush.

Let the domain of discourse be baristas and espresso drinks. Use the following definitions of predicates for this problem:

- `Barista(x)`: x is a barista
- `Espresso(x)`: x is an espresso drink
- `Strong(x)`: x has a strong flavor
- `ContainsMilk(x)`: x contains milk
- `ContainsFoam(x)`: x contains foam
- `Likes(x, y)`: x likes y

1.1. Round One [12 points]

Translate the following observations into English. Your translations should take advantage of "restricting the domain" to make more natural translations when possible, but you should not otherwise simplify the expression before translating.

Specifically, we have these requirements for translations in this problem

- You must not use variable names in your English translation (e.g., don't say "for every x...")
- For every quantified variable where one or more of the predicates can be interpreted as a domain restriction, you must use at least one of them to make your translation more natural. So with a domain of discourse of all integers, $\forall x(\texttt{Even}(x) \wedge \texttt{Prime}(x)] \rightarrow \texttt{IsEqual}(x, 2))$ could be translated as "For every even integer, if it is prime it is equal to 2" or "Every prime and even integer is equal to 2" but could not be translated as "For every integer, if the integer is prime and even then it is equal to 2."

(a) $\forall x \forall y([\texttt{Barista(x)} \wedge \texttt{Espresso(y)} \wedge \texttt{Strong(y)}] \rightarrow \texttt{Likes(x, y)})$

(b) $\exists x(\texttt{Espresso(x)} \wedge \neg\, \texttt{Strong(x)} \wedge \neg\, \texttt{ContainsFoam(x)} \wedge \neg\, \texttt{ContainsMilk(x)})$

(c) $\neg\exists x(\texttt{Espresso(x)} \wedge \texttt{ContainsFoam(x)} \wedge \neg\, \texttt{ContainsMilk(x)}) \wedge \forall x(\texttt{Espresso(x)} \rightarrow \texttt{Strong(x)})$

1.2. Round Two [4 points]

You realize that the first sentence (i.e., part a) is false. State the negation of (a) in English. You should simplify the negation so that the English sentence is natural. Simplify here mainly means that negations should be applied only to individual predicates. For example you must say "x is not prime and it is not even" rather than "it is not the case that x is prime or x is even."

2. Become a Domain Expert [10 points]

For the following statements, translate them into predicate logic (specifying and defining any predicates you use). Then provide a domain of discourse where the statement is true and another domain of discourse where the statement is false.

Also include 1-2 sentences for each domain for why the statement has the truth value it does.

If you wish to make extra assumptions about the world you may do so as long as you state them.

(a) For every a and b, there exists a c such that a is the product of b and c. Broadly, both domains of discourse you give should be some group of numbers.
Hint: Remember that you can specify your domain of discourse to exclude certain numbers or groups of numbers (e.g., "negative real numbers", "even integers", or "rational numbers except for $\frac{3}{4}$").

(b) There is an x such that for every y, x is a mammal if and only if y is a reptile. Broadly, both domains of discourse you give should be some group of animals.

3. Nested Quantifiers [15 points]

Fix your domain of discourse to be all desserts.

Use the predicates $\texttt{Cake}(x)$, $\texttt{Brownie}(x)$, $\texttt{Fruitcake(x)}$, and $\texttt{Cookie}(x)$ to say x is a cake, brownie, fruitcake, or cookie, respectively. Similarly, use the predicates $\texttt{HasFruit}(x)$ to say x has fruit. Finally, the predicate $\texttt{Tastier}(x, y)$ means x is tastier than y (note the order).

In this problem, an example of something you might give for a "scenario" might be "There is at least one cake in the domain of discourse that does not have fruit". You can assume that the given statements are true facts about desserts.

(a) Your friend tried to translate "Every fruitcake has fruit and is a cake" and got

$$\forall x(\texttt{Fruitcake}(x) \wedge [\texttt{Cake}(x) \wedge \texttt{HasFruit}(x)]).$$

The translation is incorrect. Give a correct translation, and describe a scenario in which your translation and their translation evaluate to different truth values.

(b) Your friend tried to translate "There is a cake tastier than all brownies" and got

$$\exists x \forall y([\texttt{Cake}(x) \wedge \texttt{Brownie}(y)] \rightarrow \texttt{Tastier}(x, y)).$$

The translation is incorrect. Give a correct translation, and describe a scenario in which your translation and their translation evaluate to different truth values.

(c) Translate the sentence "For every dessert d, there is a dessert e such that for every dessert f: d is tastier than e or f is tastier than e" into predicate logic.

4. Nope [12 points]

For this question, our domain of discourse is "Cats", and you may use the following predicates:

- $\texttt{Tabby}(x)$: x is a tabby cat
- $\texttt{Orange}(x)$: x is a cat with orange fur
- $\texttt{Mischief}(x)$: x is causing mischief
- $\texttt{Lasagna}(x)$: x is eating lasagna

(a) Translate the following sentence to predicate logic [3 points]:

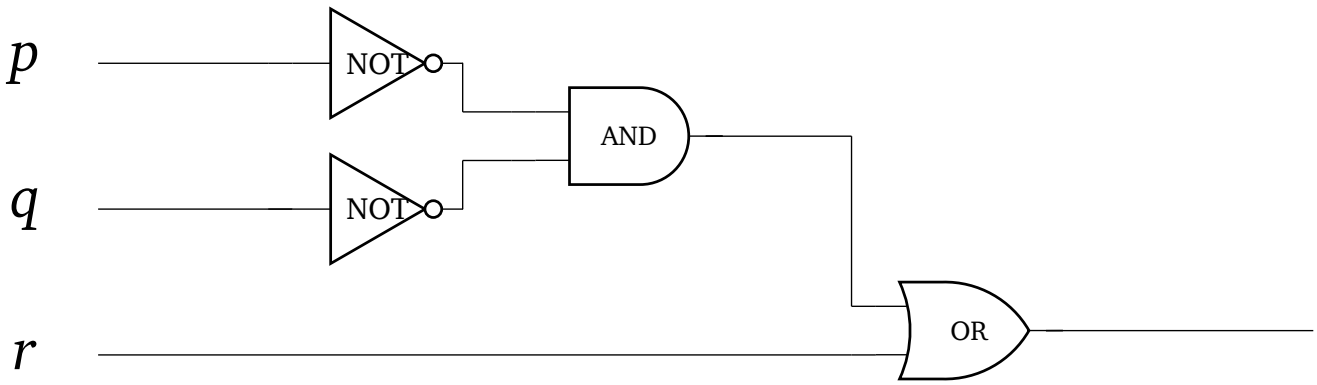
"There is an orange tabby cat, and it causes mischief unless it's eating lasagna."

(b) Negate the predicate logic sentence that you translated above in part (a); give your answer in predicate logic (i.e., symbols not in English). In your final answer, make sure that each $\neg$ symbol is applied only to individual predicates. For example instead of $\neg(\texttt{P}(x) \wedge \texttt{Q}(x))$, you should simplify to $(\neg\texttt{P}(x) \vee \neg\texttt{Q}(x))$. [5 points]
Note: For this part, a completely correct final answer will receive full credit, but we encourage you to show work so we can give partial credit.

5. Plenty of Propositional Proposals [10 points]

Submit this question online through the Gradescope assignment titled "Homework 3 Q5". You do not need to include it with the rest of your submission. You will check boxes on gradescope for this, so you shouldn't spend time making a neat typed version of this problem, but we do recommend writing out the problem before entering your answers so you can check it more easily.

(a) Fill out the truth table for the output column of the following circuit. You may find it helpful to convert to a logical expression first. [4 points]



p	q	r	OUT
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

(b) Write the DNF and CNF expressions using the table in part (a). [6 points]

6. The Truth Will Set You Free [12 points]

Below is the truth table for the propositional expression $\neg b \wedge (a \vee c)$.

a	b	c	$\neg b \wedge (a \vee c)$
T	T	T	F
T	T	F	F
T	F	T	T
T	F	F	T
F	T	T	F
F	T	F	F
F	F	T	T
F	F	F	F

The DNF of it is $(\neg b \wedge a \wedge c) \vee (\neg b \wedge a \wedge \neg c) \vee (\neg b \wedge \neg a \wedge c)$. Use propositional equivalences to show that the DNF is equivalent to the original expression.
That is, show $(\neg b \wedge a \wedge c) \vee (\neg b \wedge a \wedge \neg c) \vee (\neg b \wedge \neg a \wedge c) \equiv \neg b \wedge (a \vee c)$.

We've done the first few steps of the proof for you.

$$
\begin{aligned}
(\neg b \wedge a \wedge c) \vee (\neg b \wedge a \wedge \neg c) \vee (\neg b \wedge \neg a \wedge c) &\equiv (\neg b \wedge [a \wedge c]) \vee (\neg b \wedge [a \wedge \neg c]) \vee (\neg b \wedge [\neg a \wedge c]) && \text{Associative law 3x} \\
&\equiv \neg b \wedge ((a \wedge c) \vee (a \wedge \neg c)) \vee (\neg b \wedge [\neg a \wedge c]) && \text{Distributivity} \\
&\equiv \neg b \wedge ((a \wedge c) \vee (a \wedge \neg c) \vee (\neg a \wedge c)) && \text{Distributivity} \\
&\equiv \ldots
\end{aligned}
$$

Part 2 begins on the next page

Part 2

7. Being Direct [12 points]

Let the domain of discourse for this problem be integers. Define the predicates Odd$(x) := \exists k(x = 2k + 1)$, and Even$(x) := \exists k(x = 2k)$.

(a) Translate the following claim to predicate logic: [4 points]

For all even integers n and m, $7n + 4m$ is even.

(b) Prove that the claim is true. [8 points]
For this problem, write an inference proof (the kind we did in Lectures 7 and 8). You can use the justification "Algebra" to do algebraic operation(s) and the justification "definition of even" or "definition of odd" to apply those definitions. [8 points]

8. Oddly Even [12 points]

In this problem we will analyze the statement:

For all integers n, if $n - 13$ is odd, then n^2 must be even.

(a) Translate the claim into predicate logic. Let your domain of discourse be integers, and use the definitions in the previous problem. [4 points]

(b) Prove the claim is true by writing an English proof. [8 points]

9. Feedback [1 point]

Answer these questions on the separate gradescope box for this question.

Please keep track of how much time you spend on this homework and answer the following questions. This can help us calibrate future assignments and future iterations of the course, and can help you identify which areas are most challenging for you.

- How many hours did you spend working on this assignment (excluding any extra credit questions, if applicable)? Report your estimate to the nearest hour.
- Which problem did you spend the most time on?
- Any other feedback for us?