Proof by Contradiction Skeleton

Claim: $p$ is true.
- Suppose for the sake of contradiction $\neg p$.
- ...
- Then some statement $s$ must hold.
- ...
- And some statement $\neg s$ must hold.
- But $s$ and $\neg s$ is a contradiction. So $p$ must be true.

Graph Example

Can we travel on every road, without going on a road twice?
Another Proof by Contradiction

Claim: There are infinitely many primes
Proof:
Suppose for the sake of contradiction, there are only finitely many primes. Call them \( p_1, p_2, \ldots, p_k \).

Where can we find a contradiction?
- Show our list is non-inclusive (i.e., create a different prime number)
- Show one of the numbers in our list is not prime
- Create a contradiction with facts about prime factorization
- Show \( 1 = 2 \)
- Show \( p \) is odd and even at the same time
- Proof by cases with a mix of the above

But \( \) is a contradiction! So, there must be infinitely many primes.

Another Proof by Contradiction

Claim: There are infinitely many primes
Proof:
Suppose for the sake of contradiction, there are only finitely many primes. Call them \( p_1, p_2, \ldots, p_k \).

Consider the number \( q = p_1 \cdot p_2 \cdot \ldots \cdot p_k + 1 \)

Case 1: \( q \) is prime:

Case 2: \( q \) is not prime (i.e., composite):

Since \( q \) is composite, we know that some prime \( p_i \) must divide \( q \). This means that \( q \% p_i = 0 \).

Also, notice that \( q \% p_i = (p_1 \cdot p_2 \cdot \ldots \cdot p_k + 1) \% p_i \) using the definition of \( q \), which gives us:

\[
q \% p_i = (p_1 \cdot p_2 \cdot \ldots \cdot p_k) + 1 \% p_i
\]

In both cases, this is a contradiction! So, there must be infinitely many primes.