Another Proof

Claim: $\forall a (\text{Even}(a^2) \rightarrow \text{Even}(a))$ “if $a^2$ is even, then $a$ is even.”

See how far you get (this is somewhat a trick question).

At the very least, introduce variables, assume anything you can at the start, put down your “target” at the bottom of the paper.

Divides

For integers $x, y$ we say $x|y$ (“$x$ divides $y$”) iff there is an integer $z$ such that $xz = y$.

Which of these are true?

$2|4$  $4|2$  $2|\ -2$

$5|0$  $0|5$  $1|5$
Unique

**The Division Theorem**

For every \( a \in \mathbb{Z}, \ d \in \mathbb{Z} \) with \( d > 0 \)
There exist **unique** integers \( q, r \) with \( 0 \leq r < d \)
Such that \( a = dq + r \)

“unique” means “only one”….but be careful with how this word is used.
\( r \) is unique, **given** \( a, d \). – it still depends on \( a, d \) but once you’ve chosen \( a \) and \( d \)

“unique” is not saying \( \exists r \forall a, d \ P(a, d, r) \)
It’s saying \( \forall a, d \exists r [P(a, d, r) \land [P(a, d, x) \rightarrow x = r]] \)

Another Proof

For all integers, \( a, b, c \): Show that if \( a \nmid (bc) \) then \( a \nmid b \) or \( a \nmid c \).
Proof:
Let \( a, b, c \) be arbitrary integers, and suppose \( a \nmid (bc) \).
Then there is not an integer \( z \) such that \( az = bc \)

... 

So \( a \nmid b \) or \( a \nmid c \)