Another Direct Proof

Prove: “The product of two odd integers is odd.”
\[ \forall x \forall y \left( (\text{Odd}(x) \land \text{Odd}(y)) \rightarrow \text{Odd}(xy) \right) \]

Definitions
\[
\text{Odd}(x) := \exists k (x = 2k + 1)
\]

Yet Another Direct Proof

Prove: “The product of two square integers is square.”
\[ \forall n \forall m \left( (\text{Square}(n) \land \text{Square}(m)) \rightarrow \text{Square}(nm) \right) \]

Definitions
\[
\text{Square}(x) := \exists k (x = k^2)
\]
Two claims, two proof techniques

Suppose I claim that all square numbers are even.
That...doesn’t look right.
How do you prove me wrong?

What am I trying to prove? First write symbols for “¬(for all square numbers)”
Then ‘distribute’ the negation sign.

Proof By Cases

Claim: ∀x( Prime(x) → [ Odd(x) ∨ PowerOfTwo(x)])
Where PowerOfTwo(x) := ∃c(Integer(c) ∧ x = 2^c)
*You may assume for this proof that 2 is the only even prime.*
Let x be an arbitrary prime number.

You need two different arguments!