Another Direct Proof

Definitions
$\mathrm{Odd}(x) \coloneqq \exists k(x = 2k + 1)$

Prove: "The product of two odd integers is odd."

$$\forall x \forall y \left(\left(\mathrm{Odd}(x) \land \mathrm{Odd}(y) \right) \to \mathrm{Odd}(xy) \right)$$

Yet Another Direct Proof

Definitions
$\mathrm{Square}(x) \coloneqq \exists k \left(x = k^2 \right)$

Prove: "The product of two square integers is square."

$$\forall n \forall m \left(\left(\mathrm{Square}(n) \land \mathrm{Square}(m) \right) \to \mathrm{Square}(nm) \right)$$

Two claims, two proof techniques

Suppose I claim that all square numbers are even.

That…doesn't look right.

How do you prove me wrong?

What am I trying to prove? First write symbols for "¬(for all square numbers)"

Then 'distribute' the negation sign.

Proof By Cases

Claim: $\forall x($ `Prime`$(x) \rightarrow [$ `Odd`$(x) \vee$ `PowerOfTwo`$(x)])$

Where `PowerOfTwo`$(x) \coloneqq \exists c($`Integer`$(c) \wedge x = 2\char`^c)$

You may assume for this proof that 2 is the only even prime.

Let x be an arbitrary prime number.

You need two different arguments!