Nested Quantifiers

Let our domain of discourse be \{A, B, C, D, E\}

And our proposition \(P(x, y)\) be given by the table.

What should we look for in the table?

\[\exists x \forall y P(x, y)\]

\[\forall x \exists y P(x, y)\]

<table>
<thead>
<tr>
<th>(P(x, y))</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>T</td>
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<tr>
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<td>E</td>
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</tr>
</tbody>
</table>

Try it yourselves

Every cat loves some human. There is a cat that loves every human.

Let your domain of discourse be mammals. Use the predicates \(\text{Cat}(x), \text{Dog}(x)\), and \(\text{Loves}(x, y)\) to mean \(x\) loves \(y\).
To “limit” variables to a portion of your domain of discourse under a universal quantifier add a hypothesis to an implication.

To “limit” variables to a portion of your domain of discourse under an existential quantifier AND the limitation together with the rest of the statement.

To negate an expression with a quantifier
1. Switch the quantifier (\(\forall\) becomes \(\exists\), \(\exists\) becomes \(\forall\))
2. Negate the expression inside

1. The statement is true for every \(x\), we just want to put a name on it. \(\forall x\ (p(x) \land q(x))\) means “for every \(x\) in our domain, \(p(x)\) and \(q(x)\) both evaluate to true.”

**Universal Quantifier**

- “\(\forall x\)”
- “for each \(x\)”, “for every \(x\)”, “for all \(x\)” are common translations
- Remember: upside-down-A for All.

2. There’s some \(x\) out there that works, (but I might not know which it is, so I’m using a variable).

\(\exists x (p(x) \land q(x))\) means “there is an \(x\) in our domain, \(p(x)\) and \(q(x)\) are both true.”

**Existential Quantifier**

- “\(\exists x\)”
- “there is an \(x\)”, “there exists an \(x\)”, “for some \(x\)” are common translations
- Remember: backwards-E for Exists.