Our First Proof

$(a \wedge b) \vee (\neg a \wedge b) \vee (\neg a \wedge \neg b) \equiv (a \wedge b) \vee [(\neg a \wedge b) \vee (\neg a \wedge \neg b)]$

$\equiv (a \wedge b) \vee [\neg a \wedge (b \vee \neg b)]$

$\equiv (a \wedge b) \vee [\neg a \wedge \mathrm{T}]$

$\equiv (a \wedge b) \vee [\neg a]$

$\equiv (\neg a \vee b)$

Stay on target:
We met our intermediate goal.
Don't forget the final goal!
We want to end up at $(\neg a \vee b)$

If we apply the distribution rule,
We'd get a $(\neg a \vee b)$

Properties of Logical Connectives

These identities hold for all propositions p, q, r

- Identity
  - $p \wedge \mathrm{T} \equiv p$
  - $p \vee \mathrm{F} \equiv p$
- Domination
  - $p \vee \mathrm{T} \equiv \mathrm{T}$
  - $p \wedge \mathrm{F} \equiv \mathrm{F}$
- Idempotent
  - $p \vee p \equiv p$
  - $p \wedge p \equiv p$
- Commutative
  - $p \vee q \equiv q \vee p$
  - $p \wedge q \equiv q \wedge p$
- Associative
  - $(p \vee q) \vee r \equiv p \vee (q \vee r)$
  - $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
- Distributive
  - $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
  - $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
- Absorption
  - $p \vee (p \wedge q) \equiv p$
  - $p \wedge (p \vee q) \equiv p$
- Negation
  - $p \vee \neg p \equiv \mathrm{T}$
  - $p \wedge \neg p \equiv \mathrm{F}$
- DeMorgan's Laws
  - $\neg(p \vee q) \equiv \neg p \wedge \neg q$
  - $\neg(p \wedge q) \equiv \neg p \vee \neg q$
- Double Negation
  - $\neg\neg p \equiv p$
- Law of Implication
  - $p \rightarrow q \equiv \neg p \vee q$
- Contrapositive
  - $p \rightarrow q \equiv \neg q \rightarrow \neg p$

Converse, Contrapositive

Implication:

If it's raining, then I have my umbrella. $p \rightarrow q$

Contrapositive:

$\neg q \rightarrow \neg p$ If I don't have my umbrella, then it is not raining.

Converse:

If I have my umbrella, then it is raining. $q \rightarrow p$

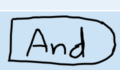
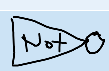
Inverse:

$\neg p \rightarrow \neg q$ If it is not raining, then I don't have my umbrella.

How do these relate to each other?

p	q	$p \rightarrow q$	$q \rightarrow p$	$\neg p$	$\neg q$	$\neg p \rightarrow \neg q$	$\neg q \rightarrow \neg p$
T	T						
T	F						
F	T						
F	F						

Meet Boolean Algebra

Name	Variables	"True/False"	"And"	"Or"	"Not"	Implication
Java Code	`boolean b`	`true,false`	`&&`	`||`	`!`	No special symbol
Propositional Logic	"p, q, r"	`T, F`	$\wedge$	$\vee$	$\neg$	$\rightarrow$
Circuits	Wires	1, 0				No special symbol
Boolean Algebra	a, b, c	1,0	$\cdot$ ("multiplication")	$+$ ("addition")	$'$ (apostrophe after variable)	No special symbol

Propositional logic

$(p \wedge q \wedge r) \vee s \vee \neg t$

Boolean Algebra

$pqr + s + t'$