Section 08: DFAs, NFAs, and Irregular Languages

1. DFAs, Stage 1

Construct DFAs to recognize each of the following languages. Let $\Sigma = \{0, 1, 2, 3\}$.

(a) All binary strings.

(b) All strings whose digits sum to an even number.

(c) All strings whose digits sum to an odd number.

2. DFAs, Stage 2

Construct DFAs to recognize each of the following languages. Let $\Sigma = \{0, 1\}$.

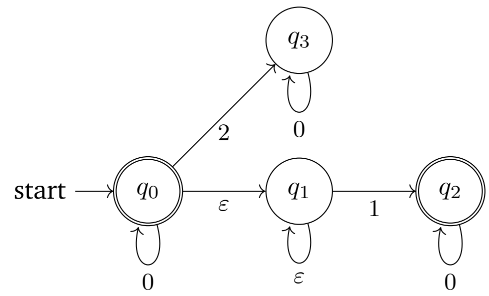
(a) All strings which do not contain the substring 101.

(b) All strings containing at least two 0's and at most one 1.

(c) All strings containing an even number of 1's and an odd number of 0's and not containing the substring 10.

3. NFAs

(a) What language does the following NFA accept?



(b) Create an NFA for the language "all binary strings that have a 1 as one of the last three digits".

4. Irregularity

(a) Let $\Sigma = \{0, 1\}$. Prove that $\{0^n 1^n 0^n \ : \ n \geq 0\}$ is not regular.

(b) Let $\Sigma = \{0, 1, 2\}$. Prove that $\{0^n (12)^m \ : \ n \geq m \geq 0\}$ is not regular.

5. Countability

Prove that the set $S := \{3x : x \in \mathbb{N}\}$ is countable.

6. Countability 2

You are a pirate. You begin in a square on a 2D grid which is infinite in all directions. In other words, wherever you are, you may move up, down, left, or right. Some single square on the infinite grid has treasure on it. Find a way to ensure you find the treasure in finitely many moves.

How can you use this idea allow you to prove that integer pairs on the 2D plane (ie. the set $\{(x, y) : x, y \in \mathbb{R}\}$) are countabley infinite? Do you see how to extend the argument to prove the rationals are countable?

7. Uncountability

Prove that $\mathcal{P}(\mathbb{N})$ is uncountable.