Section 07: Regular Languages and Number Theory

1. A Hint for the Homework

This problem uses a similar technique to the number theory problem on HW6.

Prove that if $18|(n-4)$ then $19|(2^n+3)$ for all integers $n \geq 0$.

Hint: You may use without proof the fact that $2^{18} \equiv_{19} 1$.

2. Regular Expressions

(a) Write a regular expression that matches base 10 numbers (e.g., there should be no leading zeroes).

(b) Write a regular expression that matches all base-3 numbers that are divisible by 3.

(c) Write a regular expression that matches all binary strings that contain the substring "111", but not the substring "000".

(d) Write a regular expression that matches all binary strings that do not have any consecutive 0's or 1's.

(e) Write a regular expression that matches all binary strings of the form $1^k y$, where $k \geq 1$ and $y \in \{0,1\}^*$ has at least k 1's.

3. CFGs

Write a context-free grammar to match each of these languages.

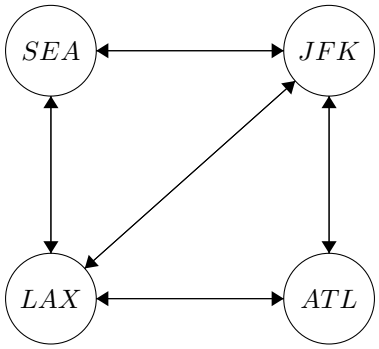
(a) All binary strings that end in 00.

(b) All binary strings that contain at least three 1's.

(c) All binary strings of the form xy, where $|x| = |y|$, but $x \neq y$.

4. Airports

Suppose you want to book a flight from Seattle (SEA) to New York (JFK), but you're not sure which route to take. In the diagram below, suppose that lines are drawn between two airports if and only if there exist regular flights between them.



Design an CFG which describes all the possible flight paths from SEA to JFK. A flight path is defined as a sequence of airport codes separated by $\rightsquigarrow$ to indicate a flight. For example, the following are valid:

- $SEA \rightsquigarrow LAX \rightsquigarrow ATL \rightsquigarrow JFK$
- $SEA \rightsquigarrow LAX \rightsquigarrow JFK$
- $SEA \rightsquigarrow JFK \rightsquigarrow ATL \rightsquigarrow LAX \rightsquigarrow JFK$

Note that a flight path may visit an airport any number of times, however it cannot traverse between airports which do not have a line between them on the map and it must begin with SEA and end with JFK. Your non-terminals should be the set $\{SEA, LAX, ATL, JFK, \rightsquigarrow\}$.