Section 01: Solutions

1. Translation

Translate each sentence into logic, using atomic propositions as necessary.

(a) Define and use the same set of atomic propositions for each of these sentences.

  (i) You will do well in this class if you study.

  (ii) You will do well in this class only if you study.

  (iii) You will do well in this class if and only if you study.

(b) If someone is cooking in the kitchen, then my dog will be there if and only if he is awake.

(c) Again define and use the same set of atomic propositions for each of these sentences. Recall from your introductory programming course, the stack data structure.

  (i) If the stack is empty, you can push but not pop.

  (ii) If the stack is full, you can pop but not push.

  (iii) If the stack is neither full nor empty, you can both push and pop.

Solution:

(a) Let $p :=$ "You do well in this class" and $q :=$ "You study".

  (i) $q \rightarrow p$

  (ii) $p \rightarrow q$

  (iii) $p \leftrightarrow q$

(b) Let $p :=$ "Someone is cooking in the kitchen", $q :=$ "My dog is in the kitchen", and $r :=$ "My dog is awake".

  (i) $p \rightarrow (q \leftrightarrow r)$

(c) Let $p :=$ "Stack is empty", $q :=$ "Stack is full", $r :=$ "Can push", and $s :=$ "Can pop".

  (i) $p \rightarrow (r \wedge \neg s)$

  (ii) $q \rightarrow (s \wedge \neg r)$

  (iii) $(\neg p \wedge \neg q) \rightarrow (r \wedge s)$

2. Reverse Translation

Consider the following atomic propositions:

$p :=$"The berries are ripe along the trail."
$q :=$"The bears have seen the berries."
$r :=$"Hiking is safe."

Translate this proposition back to English from logic:

$$p \rightarrow (\neg q \leftrightarrow r).$$

Solution:

If the berries are ripe along the trail then hiking is safe if and only if the bears have not seen the berries.

3. Review From Lecture

Remember in lecture we translated the following sentence ”Unless I go to a café or to campus, I do not drink coffee, but also I don’t go to cafés”

(a) Define atomic propositions and translate this sentence into formal logic.

(b) Now fill in the truth table for this expression

p	q	r	p ∨q	¬(p ∨q)	¬r	¬(p ∨q) →¬r	¬p	(¬(p ∨q) →¬r) ∧(¬p)
T	T	T						
T	T	F						
T	F	T						
T	F	F						
F	T	T						
F	T	F						
F	F	T						
F	F	F						

Solution:

p	q	r	p ∨q	¬(p ∨q)	¬r	¬(p ∨q) →¬r	¬p	(¬(p ∨q) →¬r) ∧(¬p)
T	T	T	T	F	F	T	F	F
T	T	F	T	F	T	T	F	F
T	F	T	T	F	F	T	F	F
T	F	F	T	F	T	T	F	F
F	T	T	T	F	F	T	T	T
F	T	F	T	F	T	T	T	T
F	F	T	F	T	F	F	T	F
F	F	F	F	T	T	T	T	T

4. Truth Tables

Write a truth table for each of these propositions:

(a) $(p \lor q) \to p$

(b) $(p \land \neg q \land r) \to r$

(c) $\neg(p \lor (q \land r)) \leftrightarrow (\neg p \lor (\neg q \land \neg r)$

Solution:

p	q	$p \lor q$	$(p \lor q) \to p$
T	T	T	T
T	F	T	T
F	T	T	F
F	F	F	T

p	q	r	$p \land \neg q \land r$	$(p \land \neg q \land r) \to r$
T	T	T	F	T
T	T	F	F	T
T	F	T	T	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

p	q	r	$q \land r$	$p \lor (q \land r)$	$\neg(p \lor (q \land r))$	$\neg p \lor (\neg q \land \neg r)$	$(\neg(p \lor (q \land r)) \leftrightarrow \neg p \lor (\neg q \land \neg r))$
T	T	T	T	T	F	F	T
T	T	F	F	T	F	F	T
T	F	T	F	T	F	F	T
T	F	F	F	T	F	T	F
F	T	T	T	T	F	T	F
F	T	F	F	F	T	T	T
F	F	T	F	F	T	T	T
F	F	F	F	F	T	T	T

Importantly, note that part b is a tautology, intuitively you could see this because if [something] $\land r$ is true, then certainly r must be true.

But also notice that part c is NOT a tautology, this demonstrates that you cannot negate an expression by just distributing the negative. We will see later in the class a rule that correctly allows us to distribute the negative.

5. Translation with Tricky Words

Translate the following sentences to logic, defining atomic propositions as necessary.

(a) In order to complete my homework it is sufficient to drink a lot of coffee.

(b) I am a student because I go to the university and pay tuition.

(c) In order to catch my flight it is necessary to be at the airport when it departs or it has been delayed.

(d) For a function to be analytic it is sufficient and necessary for it to be holomorphic. **Note:** You do not need to know what these words mean to translate them to logic!

Solution:

(a) Let $p :=$ "I complete my homework" and $q :=$ "I drink a lot of coffee", so the translation is

$$q \to p.$$

(b) Let $p :=$ "I am a student", $q :=$ "I go to the university", and $r :=$ "I pay tuition", so the translation is

$$(q \wedge r) \to p.$$

(c) Let $p :=$ "I catch my flight", $q :=$ "I am at the airport", $r :=$ "The flight has been delayed", so the translation is

$$p \to (q \vee r).$$

(d) Let $p :=$ "A function is analytic" and $q :=$ "A function is holomorphic", so the translation is

$$p \leftrightarrow q.$$

Fun fact: This is the statement of a very important theorem in *Complex Analysis*, the study of calculus in the imaginary plane (and will unfortunately have no relevance to this class).