Section 02: Domain Restriction and Translation

1. Quantifier Translation

For each part, appropriately define predicates and propositions and translate each of the sentences into logic, using quantifiers and domain restriction as necessary.

(a) (i) Each test-case catches at least one bug.

(ii) If all the test-cases are valid, then there is a bug as long as there is a test-case which fails.

(iii) There are bugs which are not caught by any test-cases.

(b) (i) At least one server is down if the website load-time on my laptop is >10ms and there is no server in diagnostic mode.

(ii) When the website load-time on my laptop is infinite either all of the servers are down or I do not have an internet connection.

2. Order of Quantifiers and Domain Restriction

(a) (i) Define some predicates and translate the following into logic, using quantifiers: Every UW student has a favorite class. *Be sure to do domain restriction!*

(ii) Exchange the order of quantifiers in English and then translate this new sentence to logic. *You will now have to perform the other type of domain restriction!*

(iii) Notice that if you naively exchange the order of quantifiers in the logic ($\forall a \exists b$ becomes $\exists b \forall a$) from your answer to part (i), you get something slightly different from (ii). It turns out these logical claims are almost equivalent but not quite. Can you prove they are not equivalent? **Hint:** Design a domain of discourse where one evaluates to false and the other to true.

3. Equivalence

Prove that the following are equivalent using a formal proof:

$$\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r).$$

4. Formal Proof

Write a formal proof that shows that if $p \vee q$, $q \rightarrow r$ and $r \rightarrow s$ all hold, then so must $\neg p \rightarrow s$.

5. Quantifier Switch

Consider the following pairs of sentences. For each pair, determine if one implies the other, if they are equivalent, or neither.

(a) $\forall x\ \forall y\ P(x, y)$ $\quad\quad$ $\forall y\ \forall x\ P(x, y)$

(b) $\exists x\ \exists y\ P(x, y)$ $\quad\quad$ $\exists y\ \exists x\ P(x, y)$

(c) $\forall x\ \exists y\ P(x, y)$ $\quad\quad$ $\forall y\ \exists x\ P(x, y)$

(d) $\forall x\ \exists y\ P(x, y)$ $\quad\quad$ $\exists x\ \forall y\ P(x, y)$

(e) $\forall x\ \exists y\ P(x, y)$ $\quad\quad$ $\exists y\ \forall x\ P(x, y)$