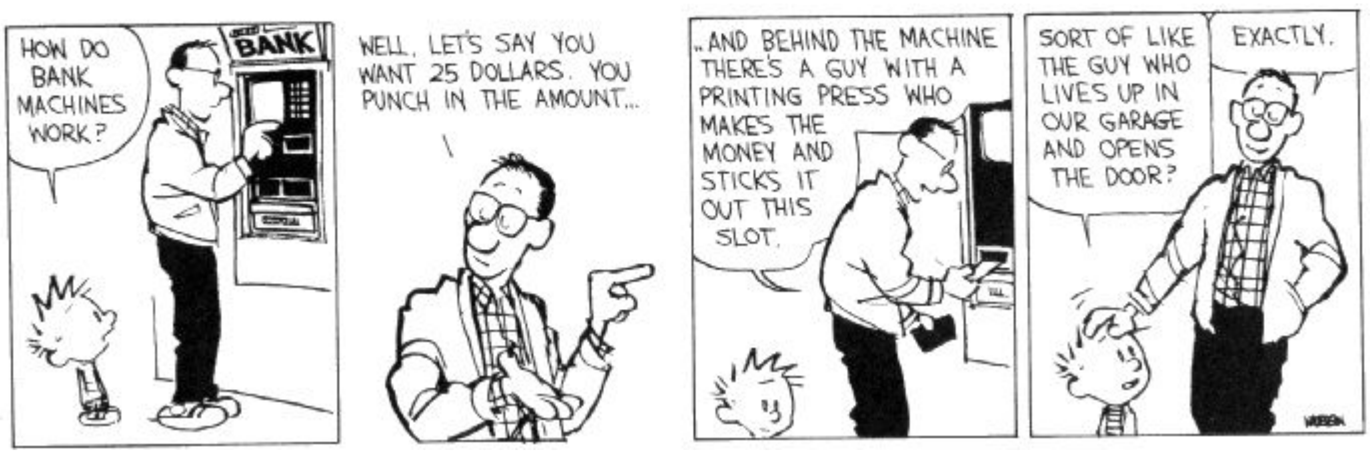
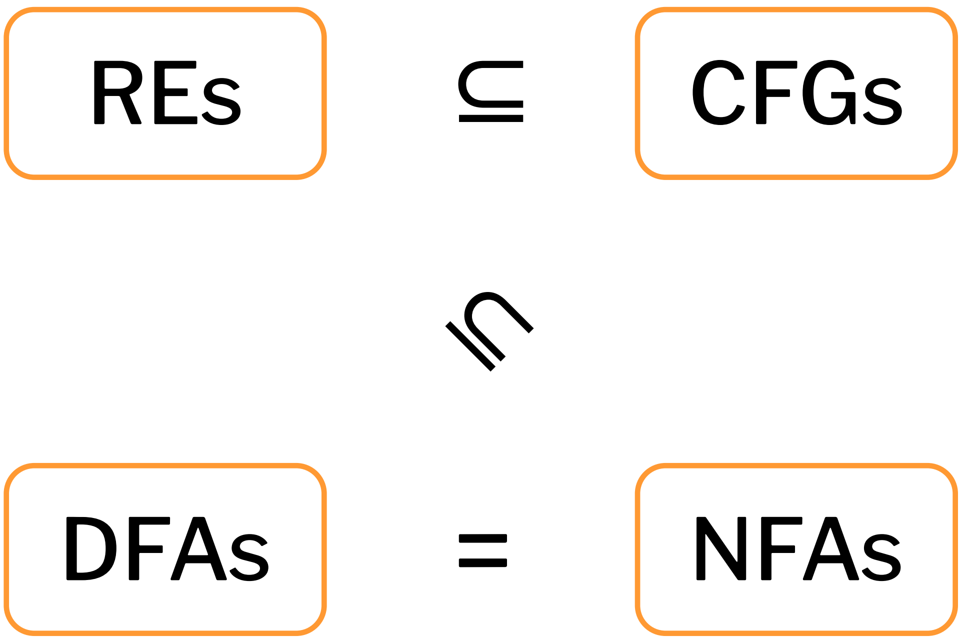
CSE 311: Foundations of Computing

Topic 10: Finite State Machines



The story so far...



Regular expressions ⊆ NFAs ≡ DFAs

We have shown how to build an optimal DFA for every regular expression

- Build NFA
- Convert NFA to DFA using subset construction
- Minimize resulting DFA

Thus, we could now implement a RegExp library

- most RegExp libraries actually simulate the NFA
- (even better: one can combine the two approaches: apply DFA minimization lazily while simulating the NFA)

The story so far...

REs $\subseteq$ CFGs

$\subseteq$

DFAs $=$ NFAs

Is this $\subseteq$ really "$=$" or "$\subsetneq$"?

Regular expressions ≡ NFAs ≡ DFAs

Theorem: For any NFA, there is a regular expression that accepts the same language

Corollary: A language is recognized by a DFA (or NFA) if and only if it has a regular expression

You need to know these facts

- the construction for the Theorem is included in the slides after this, but you will not be tested on it

New Machinery: Generalized NFAs

- Like NFAs but allow
  - parallel edges
  - regular expressions as edge labels

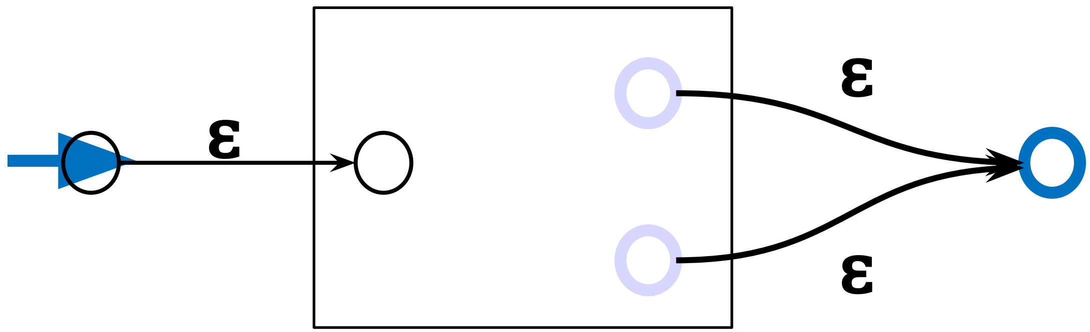
    NFAs already have edges labeled **ε** or ***a***

- The label of a path is now the concatenation of the regular expressions on those edges, making it a regular expression

- Def: A string x is accepted by a generalized NFA iff there is a *path* from start to final state labeled by a regular expression whose language **contains** x

Construction Idea

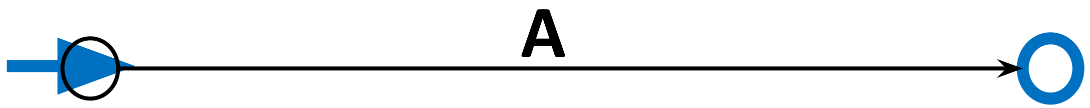
Add new start state and final state



Then delete the original states one by
one, adding edges to keep the same
language,
until the graph looks like:

Starting from an NFA

Then delete the original states one by one, adding edges to **keep the same language**, until the graph looks like:

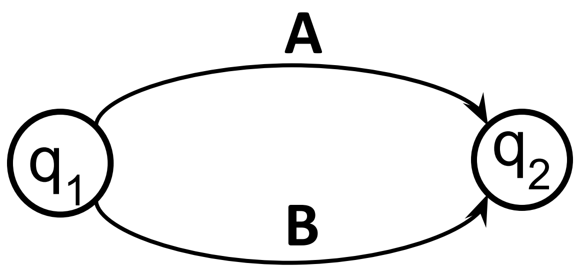


Final graph has only one path to the accepting state, which is labeled by A, so it accepts iff x is in the language of A

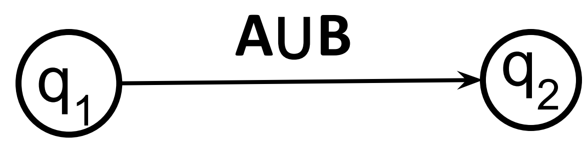
Thus, A is a regular expression with the same language as the original NFA.

Only two simplification rules

- **Rule 1**: For any two states q_1 and q_2 with parallel edges (possibly q_1=q_2), replace



by

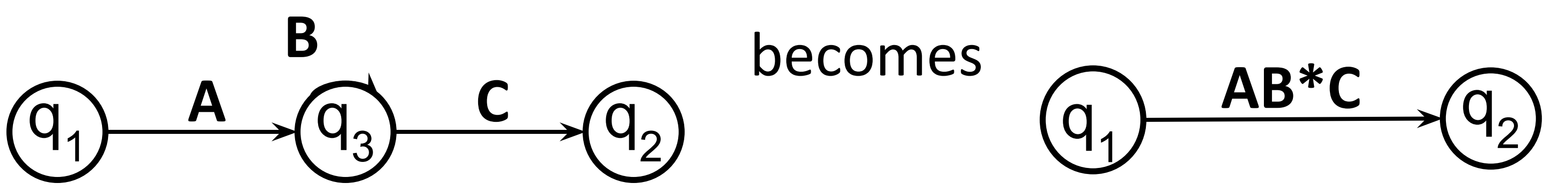


If the machine would have used the edge labeled A by consuming an input x in the language of A, it can instead use the edge labeled **AUB.**

Furthermore, this new edge does not allow transitions for any strings other than those that matched A or B.

Only two simplification rules

- **Rule 2**: Eliminate non-start/accepting state q_3 by creating direct edges that skip q_3



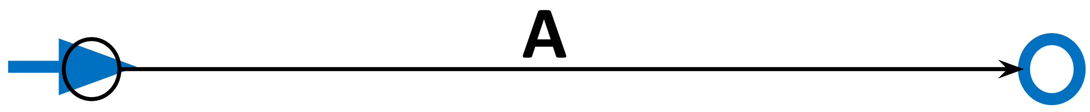
for *every* pair of states q_1, q_2 (even if $q_1=q_2$)

Any path from q_1 to q_2 would have to match AB^nC for some n (the number of times the self loop was used), so the machine can use the new edge instead. New edge *only* allows strings that were allowed before.

Construction Overview

While the box contains some state s:
for all states r, t with (r, s) and (s, t) in E:
create a direct edge (r, t) by Rule 2
delete s (no longer needed)
merge all parallel edges by Rule 1

When the loop exits, the graph looks like this:

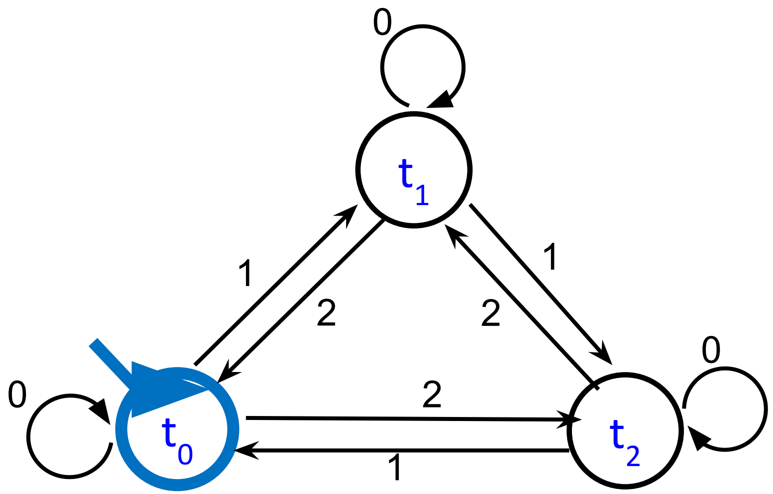


A is a regular expression with the same language as the original NFA.

Converting an NFA to a regular expression

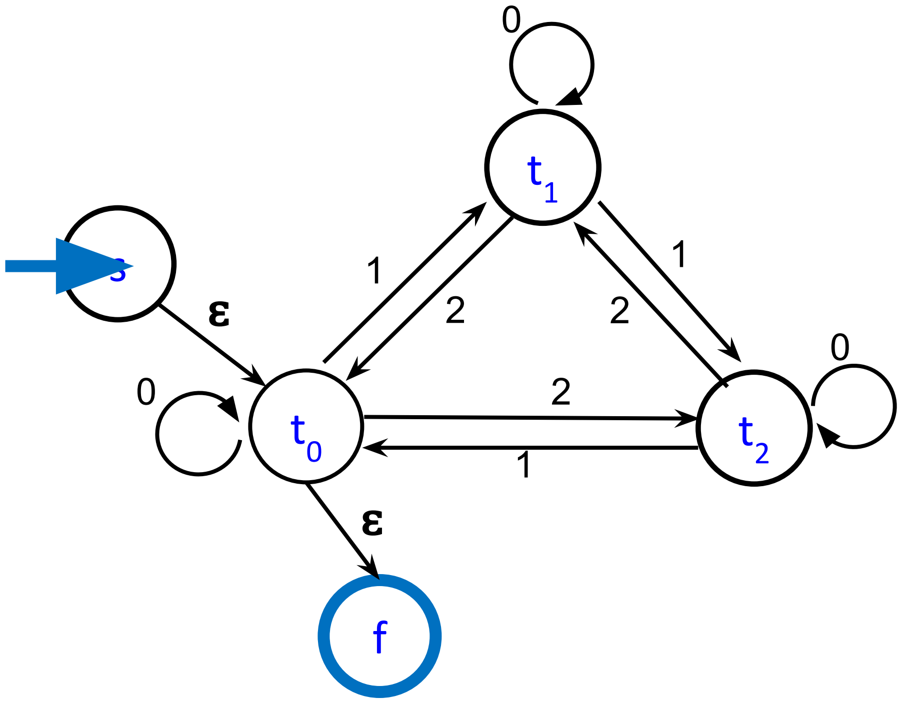
Consider the DFA for the mod 3 sum

- Accept strings from {0,1,2}* where the digits mod 3 sum of the digits is 0



Splicing out a state t_1

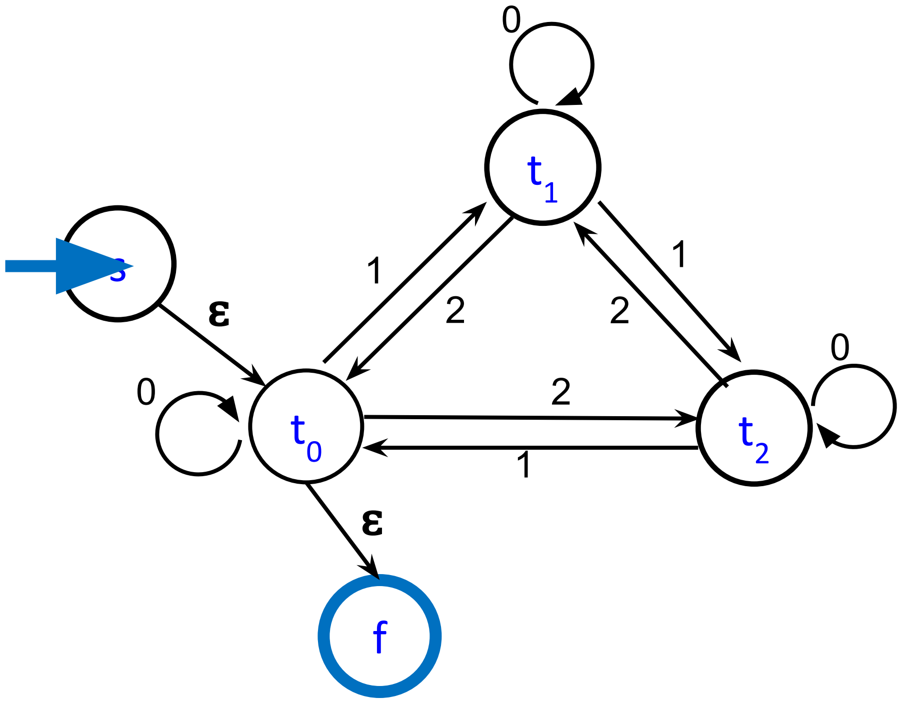
Create direct edges between neighbors of t_1 (so that we can delete it afterward)



Splicing out a state t_1

Regular expressions to add to edges

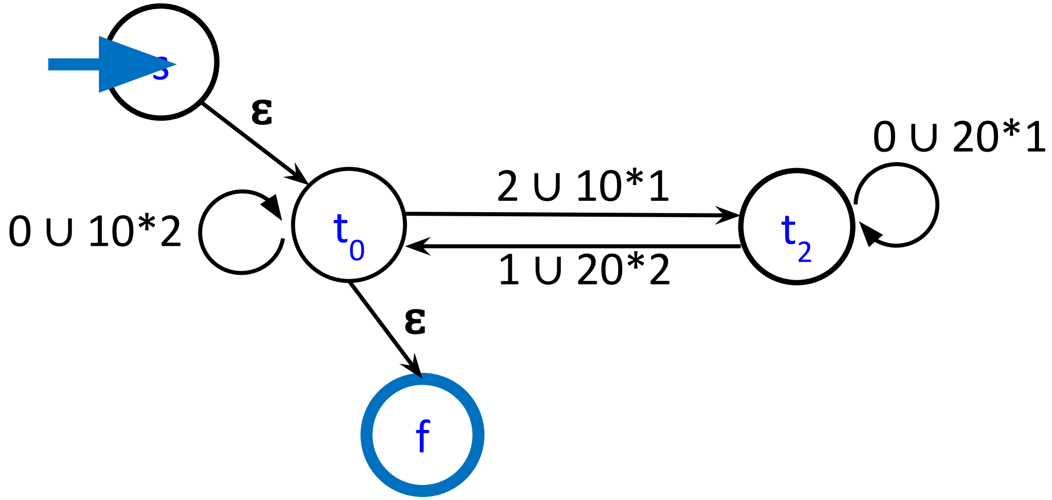
$t_0 \rightarrow t_1 \rightarrow t_0$: 10*2
$t_0 \rightarrow t_1 \rightarrow t_2$: 10*1
$t_2 \rightarrow t_1 \rightarrow t_0$: 20*2
$t_2 \rightarrow t_1 \rightarrow t_2$: 20*1



Splicing out a state t_1

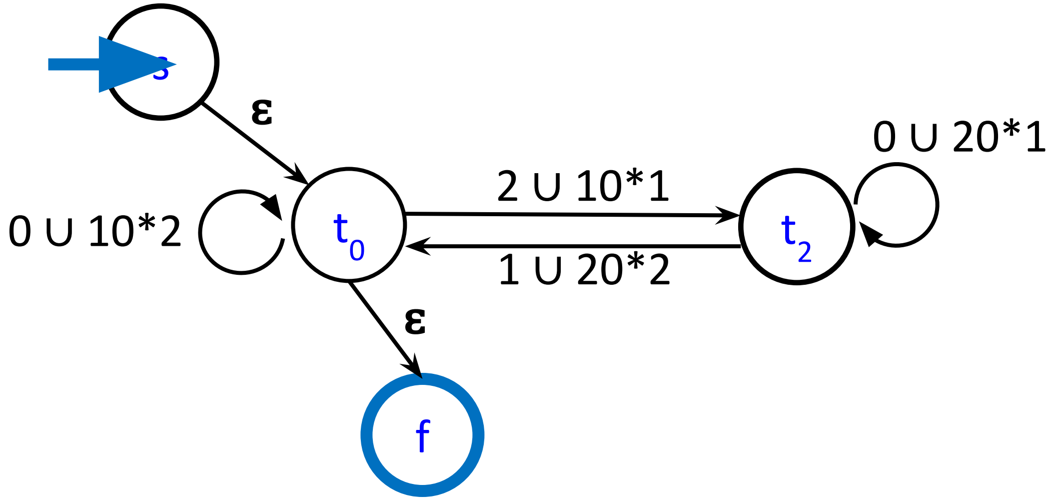
Delete t_1 now that it is redundant

$t_0 \rightarrow t_1 \rightarrow t_0$: 10*2
$t_0 \rightarrow t_1 \rightarrow t_2$: 10*1
$t_2 \rightarrow t_1 \rightarrow t_0$: 20*2
$t_2 \rightarrow t_1 \rightarrow t_2$: 20*1



Splicing out a state t_1

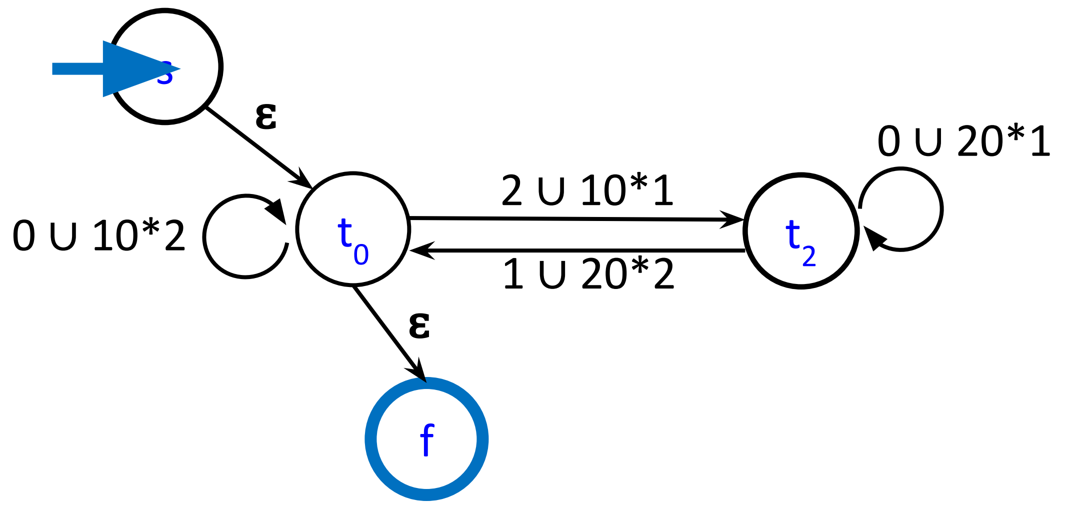
Create direct edges between neighbors of t_2
(so that we can delete it afterward)



Splicing out a state t_1

Regular expressions to add to edges

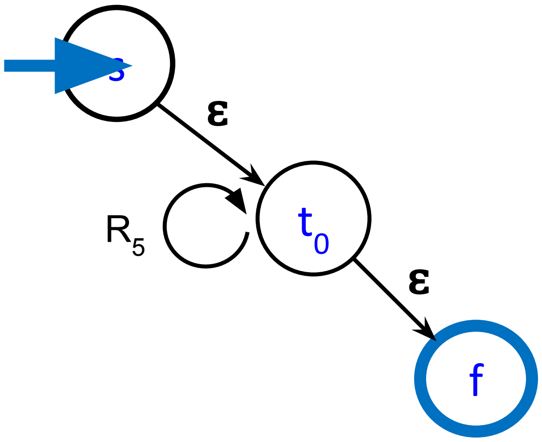
R_1: 0 ∪ 10*2
R_2: 2 ∪ 10*1
R_3: 1 ∪ 20*2
R_4: 0 ∪ 20*1



Splicing out state t_2 (and then t_0)

Delete t_2 now that it is redundant

R_1: $0 \cup 10^*2$
R_2: $2 \cup 10^*1$
R_3: $1 \cup 20^*2$
R_4: $0 \cup 20^*1$

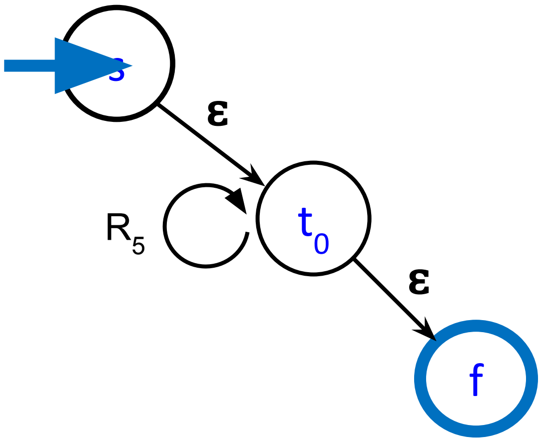


R_5: $R_1 \cup R_2R_4^*R_3$

Splicing out state t_2 (and then t_0)

Create direct (s,f) edge so we can delete t_0

R_1: $0 \cup 10^*2$
R_2: $2 \cup 10^*1$
R_3: $1 \cup 20^*2$
R_4: $0 \cup 20^*1$
R_5: $R_1 \cup R_2R_4{}^*R_3$

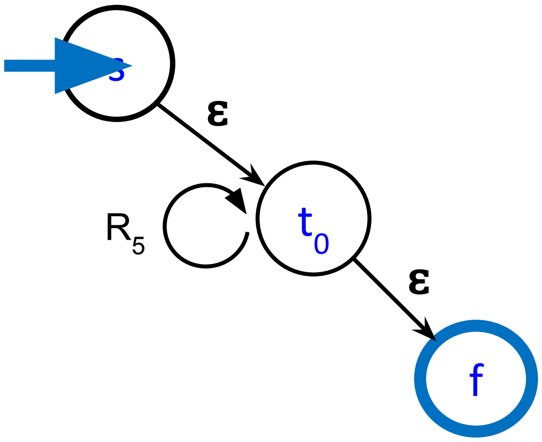


Splicing out state t_2 (and then t_0)

Regular expressions to add to edges

R_1: $0 \cup 10^*2$
R_2: $2 \cup 10^*1$
R_3: $1 \cup 20^*2$
R_4: $0 \cup 20^*1$
R_5: $R_1 \cup R_2R_4{}^*R_3$

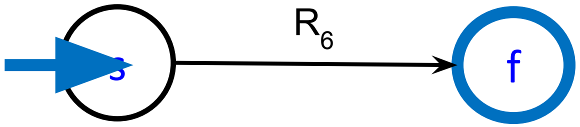
$t_0 \to t_1 \to t_0$: $R_5{}^*$



Splicing out state t_2 (and then t_0)

Delete t_0 now that it is redundant

R_1: $0 \cup 10^*2$
R_2: $2 \cup 10^*1$
R_3: $1 \cup 20^*2$
R_4: $0 \cup 20^*1$
R_5: $R_1 \cup R_2R_4{}^*R_3$

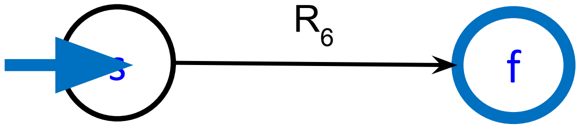


R_6: $R_5{}^*$

Splicing out state t_2 (and then t_0)

Regular expressions to add to edges

R_1: $0 \cup 10^*2$
R_2: $2 \cup 10^*1$
R_3: $1 \cup 20^*2$
R_4: $0 \cup 20^*1$
R_5: $R_1 \cup R_2R_4{}^*R_3$
R_6: $R_5{}^*$



Final regular expression: R_6 =
$(0 \cup 10^*2 \cup (2 \cup 10^*1)(0 \cup 20^*1)^*(1 \cup 20^*2))^*$

The story so far...

REs $\subseteq$ CFGs

$\equiv$

DFAs $\equiv$ NFAs

Languages represented by DFA, NFAs, or regular expressions
are called **Regular Languages**

Recall: Algorithms for Regular Languages

We have seen algorithms for

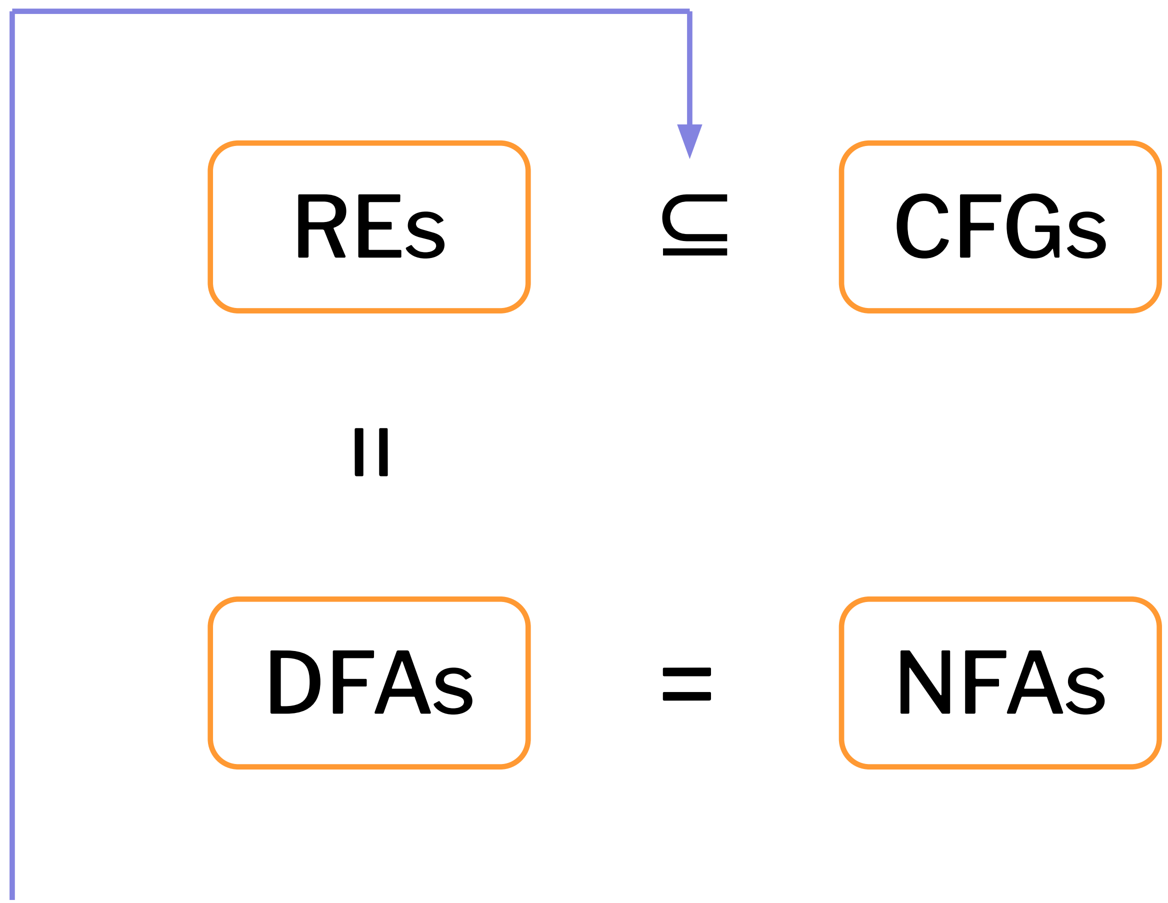
- RE to NFA
- NFA to DFA
- DFA/NFA to RE
- DFA minimization

Example Corollary of These Results

Corollary: If $\mathbf{A}$ is the language of a regular expression, then $\overline{\mathbf{A}}$ is the language of a regular expression*.

(This is the complement with respect to the universe of all strings over the alphabet, i.e., $\overline{\mathbf{A}} = \mathbf{\Sigma}^* \setminus \mathbf{A}$.)

The story so far...

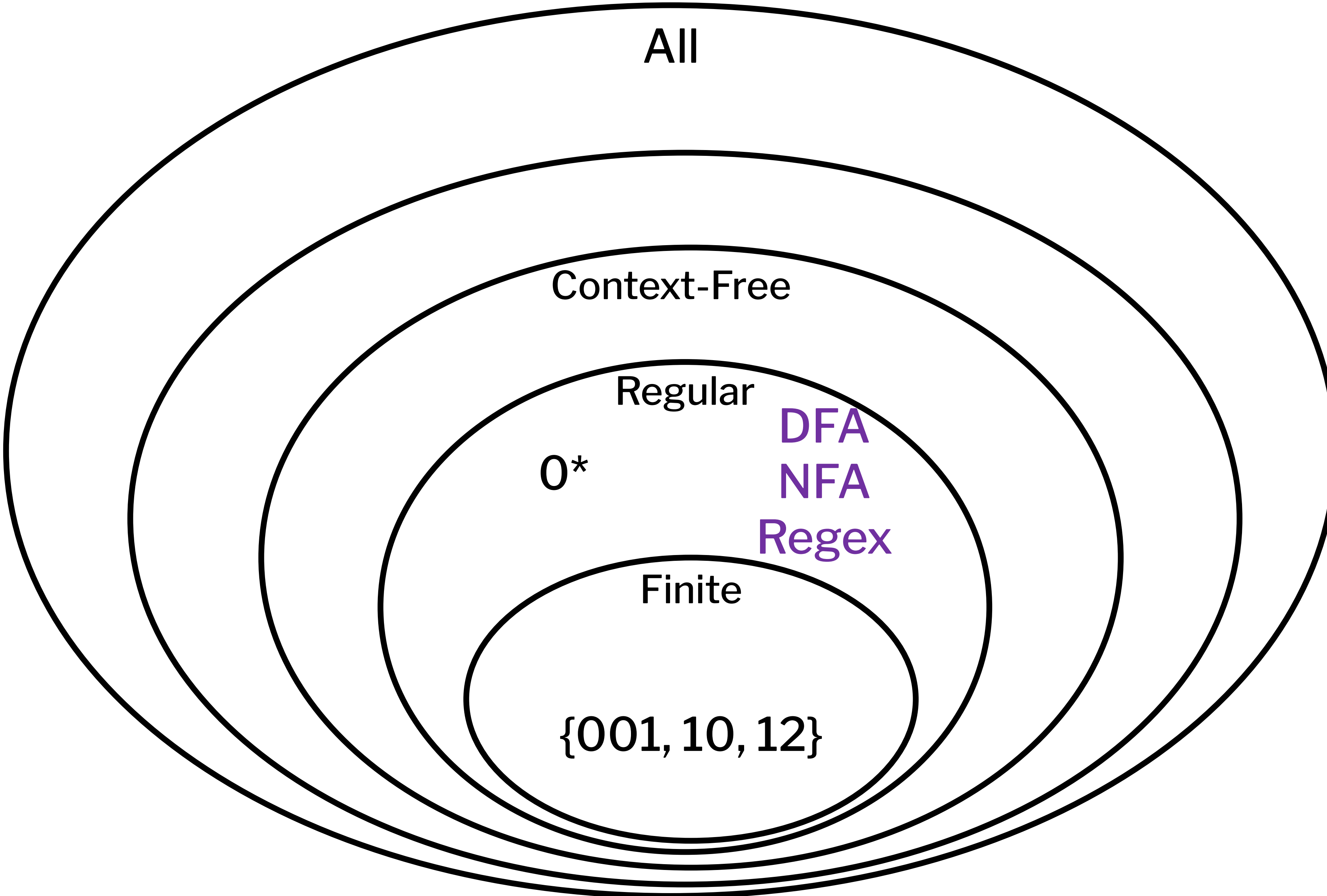


<u>Now</u>: Is this $\subseteq$ really "$=$" or "$\subsetneq$"?

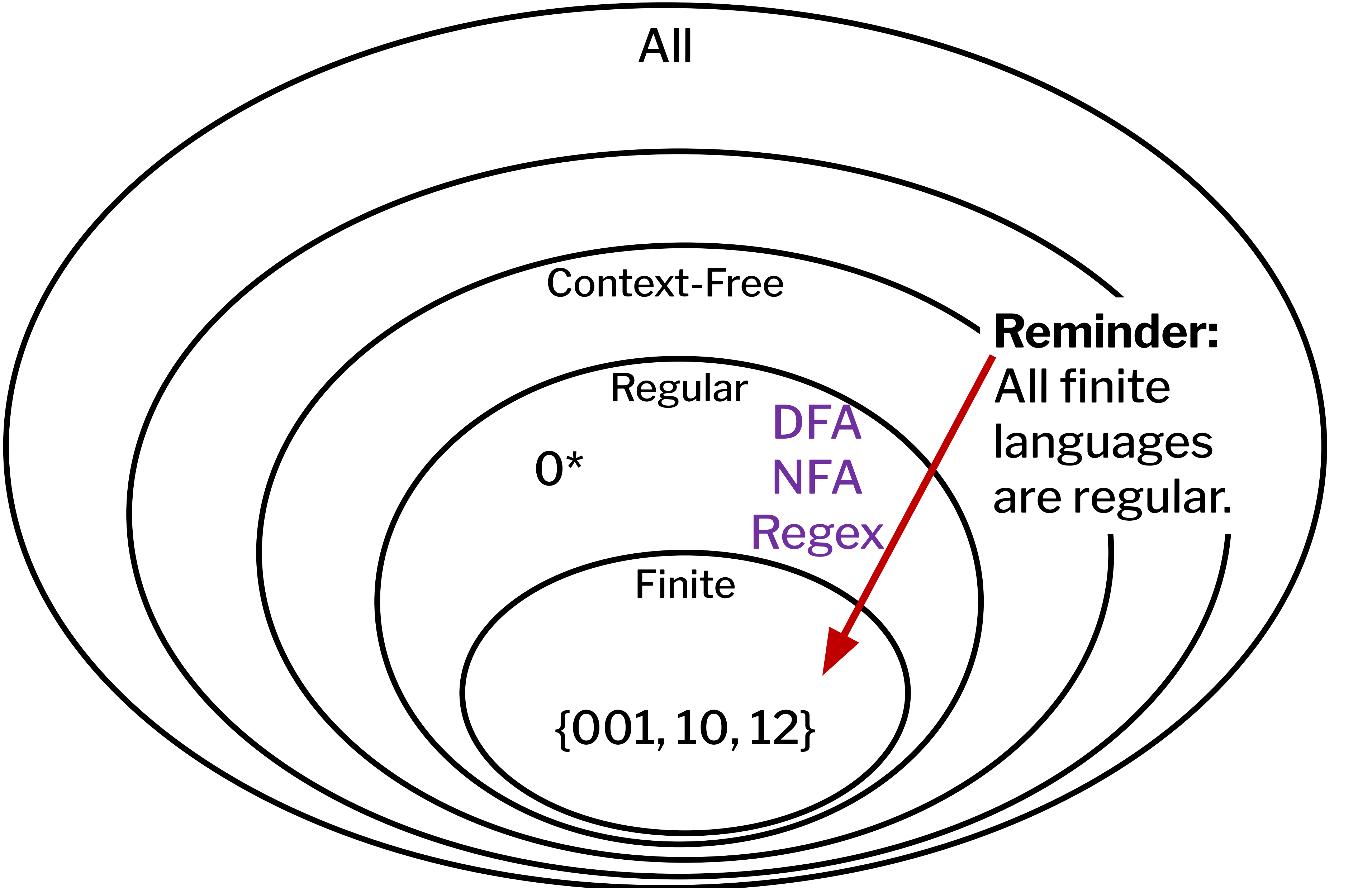
What languages have DFAs? CFGs?

All of
them?

Languages and Representations!



Languages and Representations!

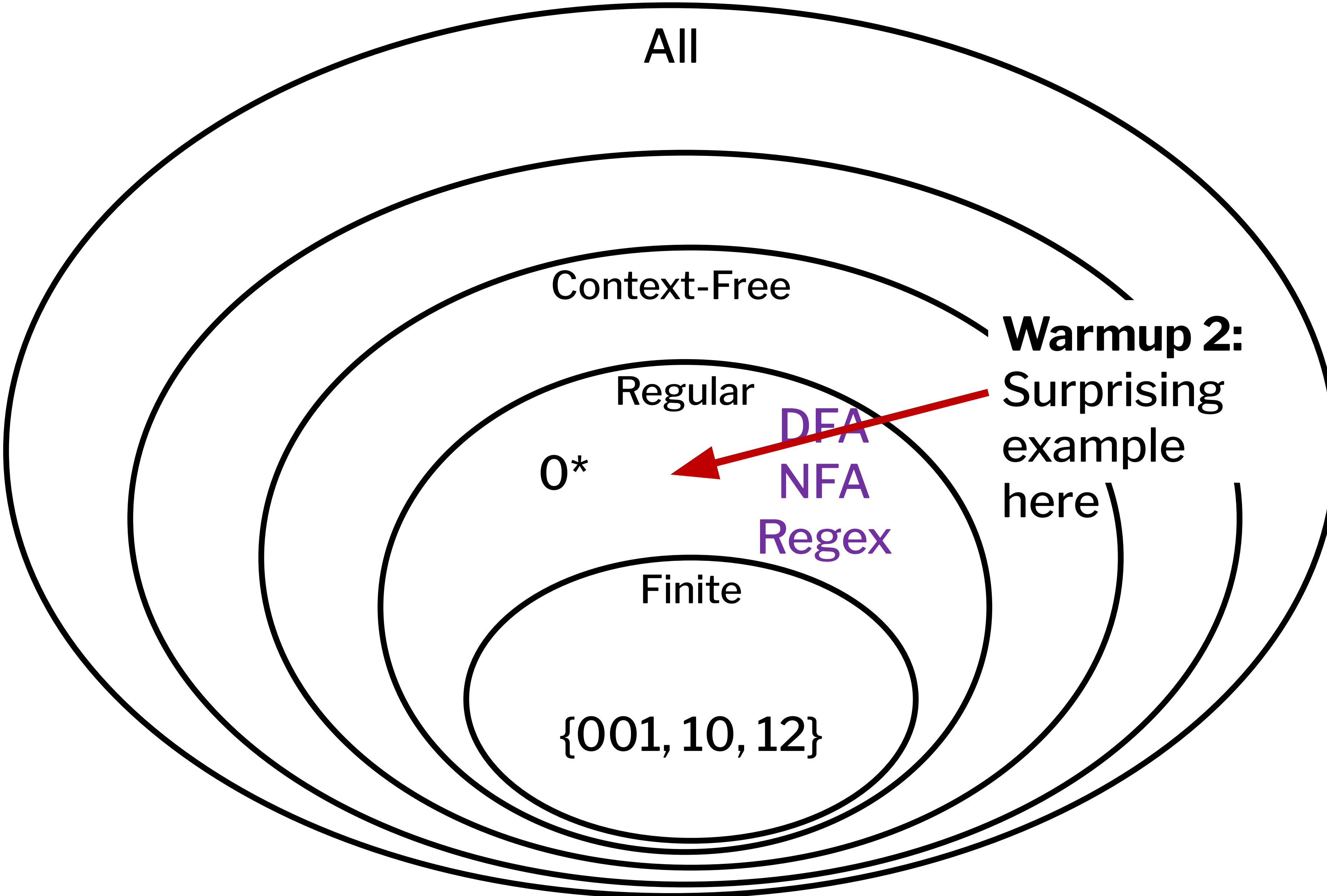


DFAs Recognize Any Finite Language

Construct a DFA for each string in the language.

Then, put them together using the union construction.

Languages and Machines!



An Interesting Infinite Regular Language

L = {**x∈** {0, 1}***: x** has an equal number of substrings 01 and 10}.

L is infinite.

0, 00, 000, …

L is regular. How could this be?

That seems to require comparing counts...

- easy for a CFG
- but seems hard for DFAs!

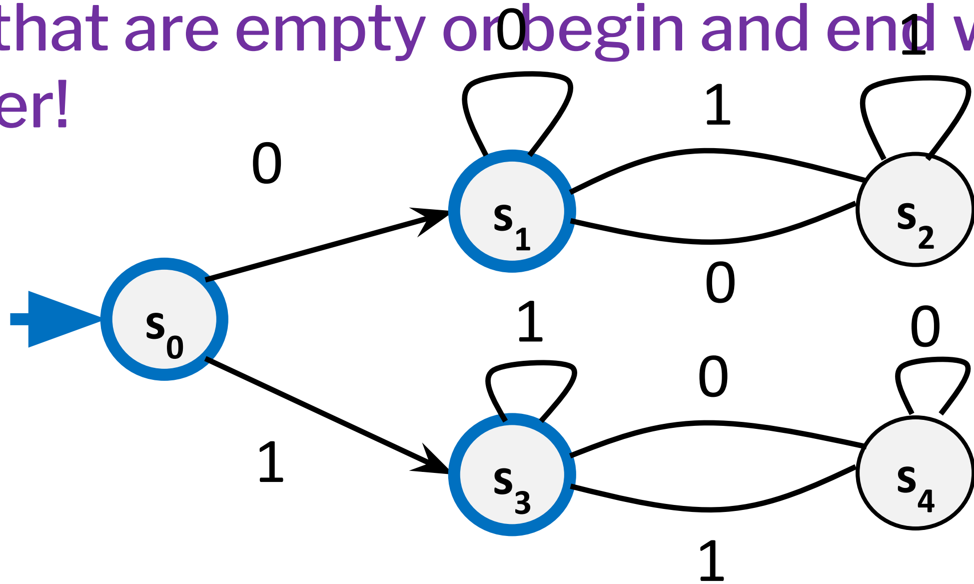
An Interesting Infinite Regular Language

L = {x∈ {0, 1}*: x has an equal number of substrings 01 and 10}.

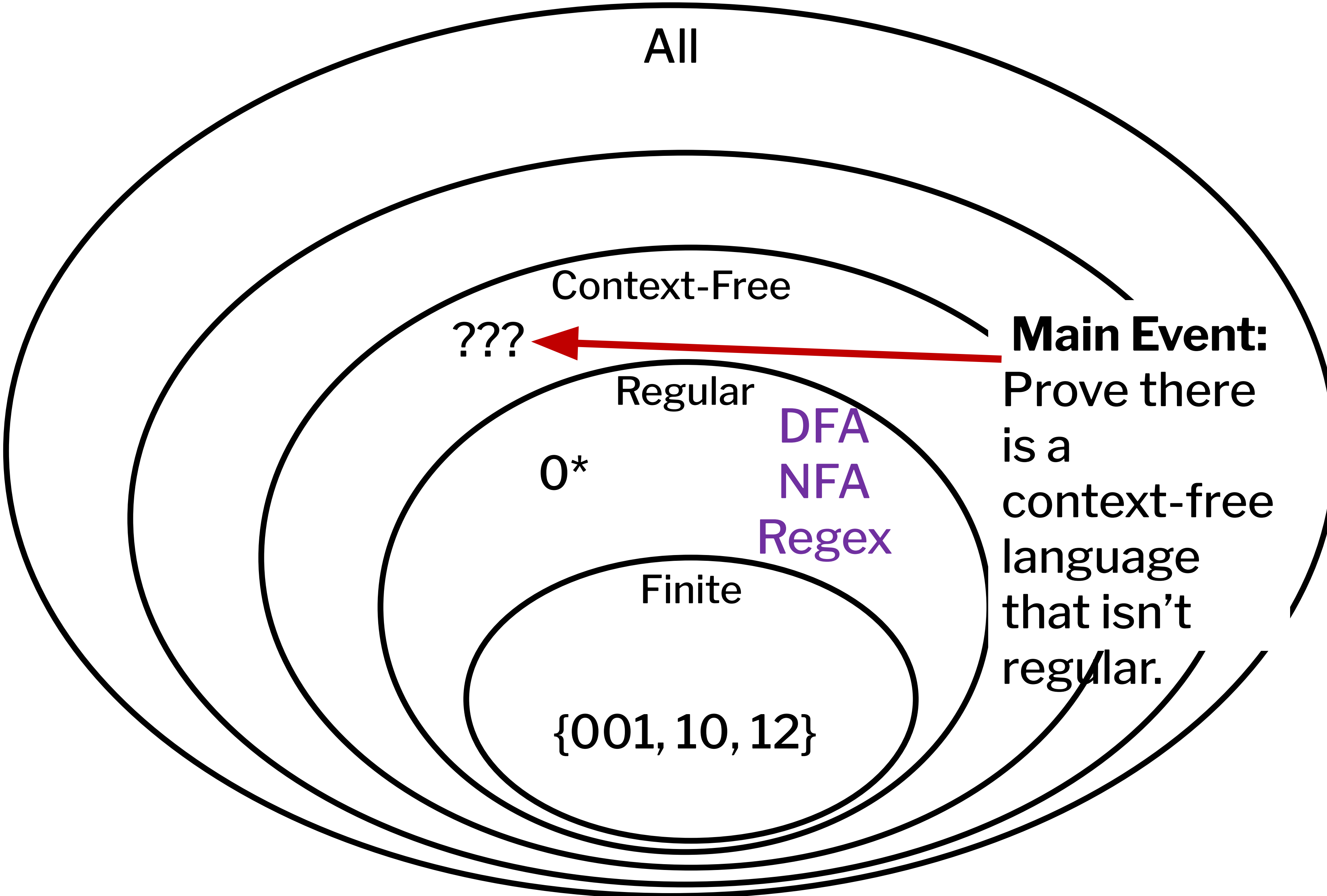
L is infinite.

0, 00, 000, ...

L is regular. How could this be? It is just the set of binary strings that are empty or begin and end with the same character!



Languages and Representations!

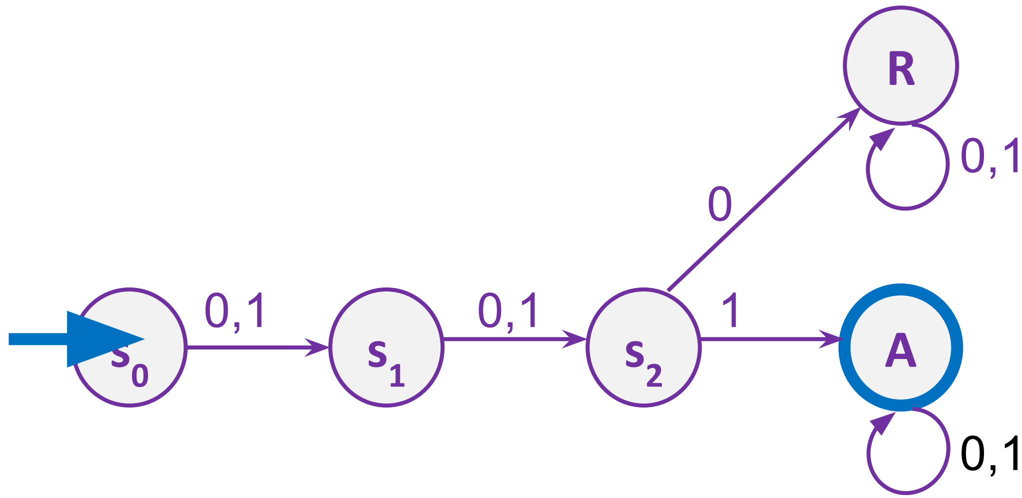


Main Event: Prove there is a context-free language that isn't regular.

Tangent: How to prove a DFA minimal?

- Show there is no smaller DFA…
- Find a set of strings that *must* be distinguished
  - Such a set is a lower bound on the DFA size

Recall: Binary strings with a 1 in the 3rd position from the start



Distinguishing set:

{ε, 0, 00, 000, 001}

The language of “Binary Palindromes” is Context-Free

-

$$S \rightarrow \varepsilon \mid 0 \mid 1 \mid 0S0 \mid 1S1$$

Is the language of “Binary Palindromes” Regular ?

Intuition (NOT A PROOF!):

Q: What would a DFA need to keep track of to decide?

A: It would need to keep track of the “first part” of the input in order to check the second part against it

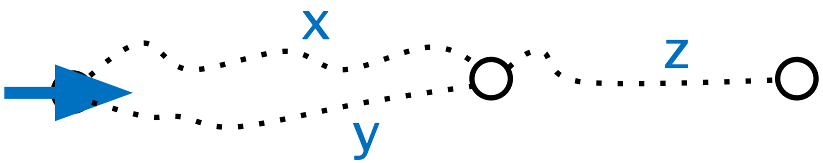
…but there are an infinite # of possible first parts and we only have finitely many states.

Proof idea: any machine that does not remember the entire first half will be wrong for some inputs

Useful Lemmas about DFAs

Lemma 1: If DFA **M** takes $\mathbf{x}, \mathbf{y} \in \Sigma^*$ to the same state, then for every $\mathbf{z} \in \Sigma^*$, M accepts $\mathbf{x} \bullet \mathbf{z}$ iff it accepts $\mathbf{y} \bullet \mathbf{z}$.

M can't remember that the input was **x**, not **y**.



$x \bullet z = x_1 x_2 \ldots x_n z_1 z_2 \ldots z_k$

$y \bullet z = y_1 y_2 \ldots y_m z_1 z_2 \ldots z_k$

Useful Lemmas about DFAs

Lemma 2: If DFA **M** has **n** states and a set **S** contains *more* than **n** strings, then **M** takes at least two strings from **S** to the same state.

M can't take n+1 or more strings to different states because it doesn't have n+1 different states.

So, some pair of strings must go to the same state.

B = {binary palindromes} can't be recognized by any DFA

Suppose for contradiction that some DFA, M, recognizes B.

We will show M accepts or rejects a string it shouldn't.

Consider S = {1, 01, 001, 0001, 00001, ...} = $\{0^n 1 : n \geq 0\}$.

B = {binary palindromes} can't be recognized by any DFA

Suppose for contradiction that some DFA, M, accepts B.

We will show M accepts or rejects a string it shouldn't.

Consider $S = \{1, 01, 001, 0001, 00001, \ldots\} = \{0^n1 : n \geq 0\}$.

Since there are finitely many states in M and infinitely many strings in S, by Lemma 2, there exist strings $0^a1 \in S$ and $0^b1 \in S$ with $a \neq b$ that end in the same state of M.

SUPER IMPORTANT POINT: You do not get to choose what a and b are. Remember, we've just proven they exist…we must take the ones we're given!

B = {binary palindromes} can't be recognized by any DFA

Suppose for contradiction that some DFA, M, accepts B.

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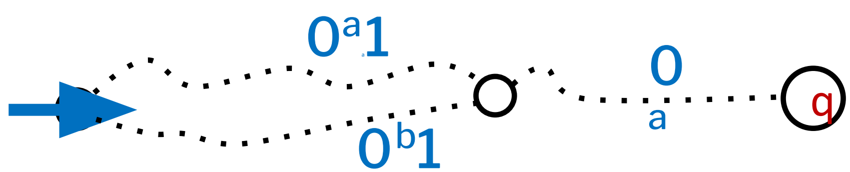
Since there are finitely many states in M and infinitely many strings in S, by Lemma 2, there exist strings $0^a1 \in S$ and $0^b1 \in S$ with $a \neq b$ that end in the same state of M.

Now, consider appending 0^a to both strings.



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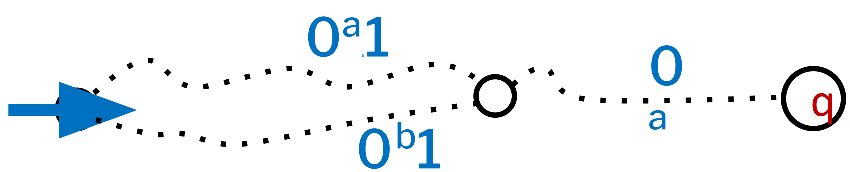

Since 0^a1 and 0^b1 end in the same state, 0^a10^a and 0^b10^a also end in the same state, call it q. But then M makes a mistake: q needs to be an accept state since $0^a10^a \in B$, but M would accept $0^b10^a \notin B$, which is an error.

B = {binary palindromes} can't be recognized by any DFA

Suppose for contradiction that some DFA, M, accepts B.

We will show M accepts or rejects a string it shouldn't.

Consider $S = \{1, 01, 001, 0001, 00001, \ldots\} = \{0^n1 : n \geq 0\}$.

Since there are finitely many states in M and infinitely many strings in S, by Lemma 2, there exist strings $0^a1 \in S$ and $0^b1 \in S$ with $a \neq b$ that end in the same state of M.

Now, consider appending 0^a to both strings.

Since 0^a1 and 0^b1 end in the same state, 0^a10^a and 0^b10^a also end in the same state, call it q. But then M makes a mistake: q needs to be an accept state since $0^a10^a \in B$, but M would accept $0^b10^a \notin B$, which is an error.

This proves that M does not recognize B, contradicting our assumption that it does. Thus, no DFA recognizes B.

Showing that a Language L is not regular

1. "Suppose for contradiction that some DFA **M** recognizes L."
2. Consider an **INFINITE** set S of prefixes (which we intend to complete later).
3. "Since **S** is infinite and **M** has finitely many states, there must be two strings s_a and s_b in **S** for $s_a \neq s_b$ that end up at the same state of **M**."
4. Consider appending the (correct) completion **t** to each of the two strings.
5. "Since s_a and s_b both end up at the same state of **M**, and we appended the same string **t**, both $s_a t$ and $s_b t$ end at the same state q of **M**. Since $s_a t \in L$ and $s_b t \notin L$, **M** does not recognize **L**."
6. "Thus, no DFA recognizes **L**."

Showing that a Language L is not regular

The choice of S is the creative part of the proof

You must find an infinite set S with the property that *no two* strings can be taken to the same state

- i.e., for ***every pair*** of strings S there is an "accept" completion that the two strings DO NOT SHARE

Prove **A** = $\{0^n1^n : n \geq 0\}$ is not regular

Suppose for contradiction that some DFA, M, recognizes **A**.

Let S =

Prove **A** = $\{0^n1^n : n \geq 0\}$ is not regular

Suppose for contradiction that some DFA, M, recognizes **A**.

Let S = $\{0^n : n \geq 0\}$. Since S is infinite and M has finitely many states, there must be two strings, 0^a and 0^b for some $a \neq b$ that end in the same state in M.

Prove $A = \{0^n 1^n : n \geq 0\}$ is not regular

Suppose for contradiction that some DFA, M, recognizes A.

Let $S = \{0^n : n \geq 0\}$. Since S is infinite and M has finitely many states, there must be two strings, 0^a and 0^b for some $a \neq b$ that end in the same state in M.

Consider appending 1^a to both strings.

Prove A = $\{0^n1^n : n \geq 0\}$ is not regular

Suppose for contradiction that some DFA, M, recognizes **A**.

Let S = $\{0^n : n \geq 0\}$. Since S is infinite and M has finitely many states, there must be two strings, 0^a and 0^b for some $a \neq b$ that end in the same state in M.

Consider appending 1^a to both strings.

Note that $0^a1^a \in$ **A**, **but** $0^b1^a \notin$ **A** since $a \neq b$. But they both end up in the same state of M, call it **q**. Since $0^a1^a \in$ **A**, state **q** must be an accept state but then M would incorrectly accept $0^b1^a \notin$ **A** so M does not recognize **A**.

Thus, no DFA recognizes A.

Prove P = {balanced parentheses} is not regular

Suppose for contradiction that some DFA, M, accepts P.

Let S =

Prove P = {balanced parentheses} is not regular

Suppose for contradiction that some DFA, M, recognizes P.

Let S = { $(^n$: n ≥ 0}. Since S is infinite and M has finitely many states, there must be two strings, $(^a$ and $(^b$ for some a ≠ b that end in the same state in M.

Prove P = {balanced parentheses} is not regular

Suppose for contradiction that some DFA, M, recognizes P.

Let $S = \{ (^n : n \geq 0\}$. Since S is infinite and M has finitely many states, there must be two strings, $(^a$ and $(^b$ for some $a \neq b$ that end in the same state in M.

Consider appending $)^a$ to both strings.

Prove P = {balanced parentheses} is not regular

Suppose for contradiction that some DFA, M, recognizes P.

Let $S = \{ (^n : n \geq 0\}$. Since S is infinite and M has finitely many states, there must be two strings, $(^a$ and $(^b$ for some $a \neq b$ that end in the same state in M.

Consider appending $)^a$ to both strings.

Note that $(^a)^a \in$ **P**, **but** $(^b)^a \notin$ **P** since $a \neq b$. But they both end up in the same state of M, call it **q**. Since $(^a)^a \in$ **P**, state **q** must be an accept state but then M would incorrectly accept $(^b)^a \notin$ **P** so M does not recognize **P**.

Thus, no DFA recognizes P.

Showing that a Language L is not regular

1. “Suppose for contradiction that some DFA **M** recognizes L.”
2. Consider an **INFINITE** set S of prefixes (which we intend to complete later). It is imperative that for ***every pair*** of strings in our set there is an “accept” completion that the two strings DO NOT SHARE.
3. “Since **S** is infinite and **M** has finitely many states, there must be two strings s_a and s_b in **S** for $s_a \neq s_b$ that end up at the same state of **M**.”
4. Consider appending the (correct) completion **t** to each of the two strings.
5. “Since s_a and s_b both end up at the same state of **M**, and we appended the same string **t**, both $s_a t$ and $s_b t$ end at the same state q of **M**. Since $s_a t \in L$ and $s_b t \notin L$, **M** does not recognize **L**.”
6. “Thus, no DFA recognizes L.”

Fact: This method is optimal

- Suppose that for a language L, the set S is a *largest* set of prefixes with the property that, for every pair $s_a \neq s_b \in S$, there is some string t such that one of $s_a t$, $s_b t$ is in L but the other isn't.
- If S is infinite, then L is not regular
- If S is finite, then the minimal DFA for L has precisely $|S|$ states, one reached by each member of S.

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- If S is infinite, then L is not regular
- If S is finite, then the minimal DFA for L has precisely $|S|$ states, one reached by each member of S.

Corollary: Our minimization algorithm was correct.

– we separated *exactly* those states for which some **t** would make one accept and another not accept

Important Notes

- It is not necessary for our strings xz with $x \in L$ to allow any string in the language
  - we only need to find a small "core" set of strings that must be distinguished by the machine

- It is not true that, if L is irregular and $L \subseteq U$, then U is irregular!
  - we always have $L \subseteq \Sigma^*$ and Σ^* is regular!
  - our argument needs different answers: $xz \in L \not\leftrightarrow yz \in L$

    for Σ^*, both strings are always in the language

Do not claim in your proof that, because $L \subseteq U$, U is also irregular

New Machinery: Generalized NFAs

- Like NFAs but allow
  - parallel edges (between the same pair of states)
  - regular expressions as edge labels

    NFAs already have edges labeled **ε** or ***a***

- Machine can follow an edge labeled by **A** by reading a <u>string of input characters</u> in the language of A
  - (if A is ***a*** or **ε**, this matches the original definition, but we now allow REs built with recursive steps.)

New Machinery: Generalized NFAs

- Like NFAs but allow
  - parallel edges
  - regular expressions as edge labels

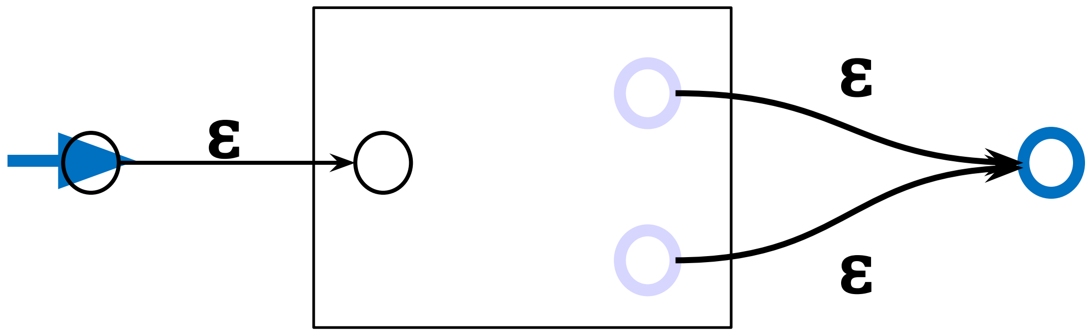
    NFAs already have edges labeled **ε** or ***a***

- The label of a path is now the concatenation of the regular expressions on those edges, making it a regular expression

- Def: A string x is accepted by a generalized NFA iff there is a *path* from start to final state labeled by a regular expression whose language **contains** x

Construction Idea

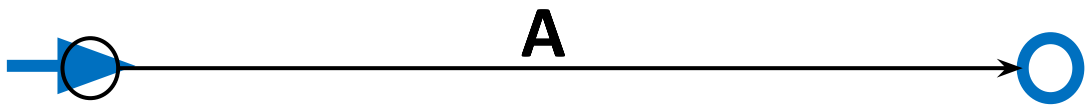
Add new start state and final state



Then delete the original states one by one, adding edges to keep the same language,
until the graph looks like:

Starting from an NFA

Then delete the original states one by one, adding edges to **keep the same language**, until the graph looks like:

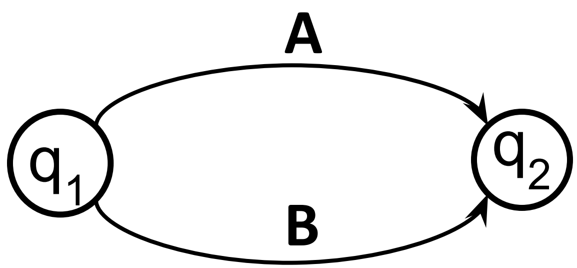


Final graph has only one path to the accepting state, which is labeled by A, so it accepts iff x is in the language of A

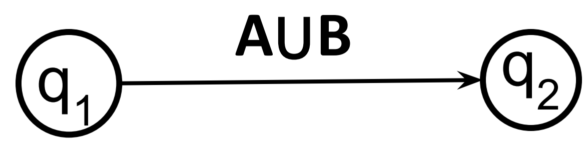
Thus, A is a regular expression with the same language as the original NFA.

Only two simplification rules

- **Rule 1**: For any two states q_1 and q_2 with parallel edges (possibly $q_1=q_2$), replace



by

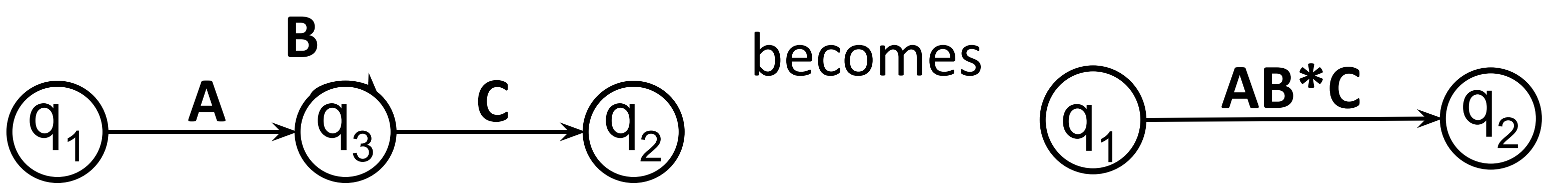


If the machine would have used the edge labeled A by consuming an input x in the language of A, it can instead use the edge labeled **AUB.**

Furthermore, this new edge does not allow transitions for any strings other than those that matched A or B.

Only two simplification rules

- **Rule 2**: Eliminate non-start/accepting state q_3 by creating direct edges that skip q_3

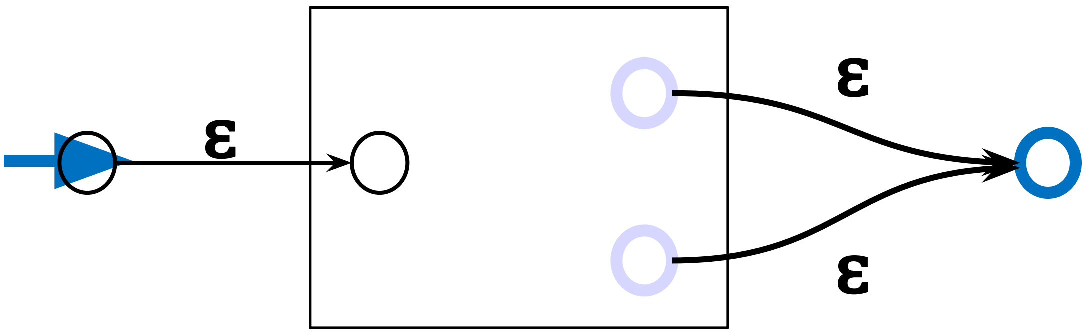


for *every* pair of states q_1, q_2 (even if q_1=q_2)

Any path from q_1 to q_2 would have to match AB^nC for some n (the number of times the self loop was used), so the machine can use the new edge instead. New edge *only* allows strings that were allowed before.

Construction Overview

Add new start state and final state



While the box contains some state s:

for all states r, t with (r, s) and (s, t) in E:

create a direct edge (r, t) by Rule 2

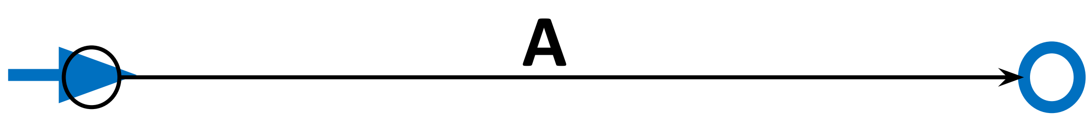
delete s (no longer needed)

merge all parallel edges by Rule 1

Construction Overview

While the box contains some state s:
  for all states r, t with (r, s) and (s, t) in E:
    create a direct edge (r, t) by Rule 2
  delete s (no longer needed)
  merge all parallel edges by Rule 1

When the loop exits, the graph looks like this:

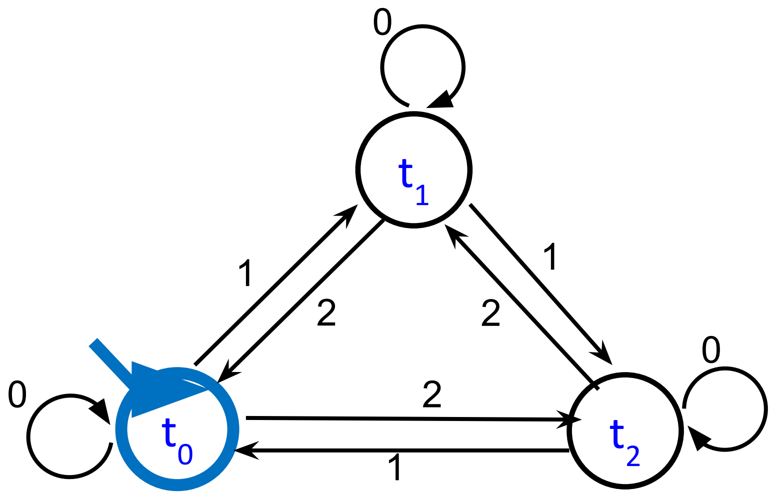


A is a regular expression with the same language as the original NFA.

Converting an NFA to a regular expression

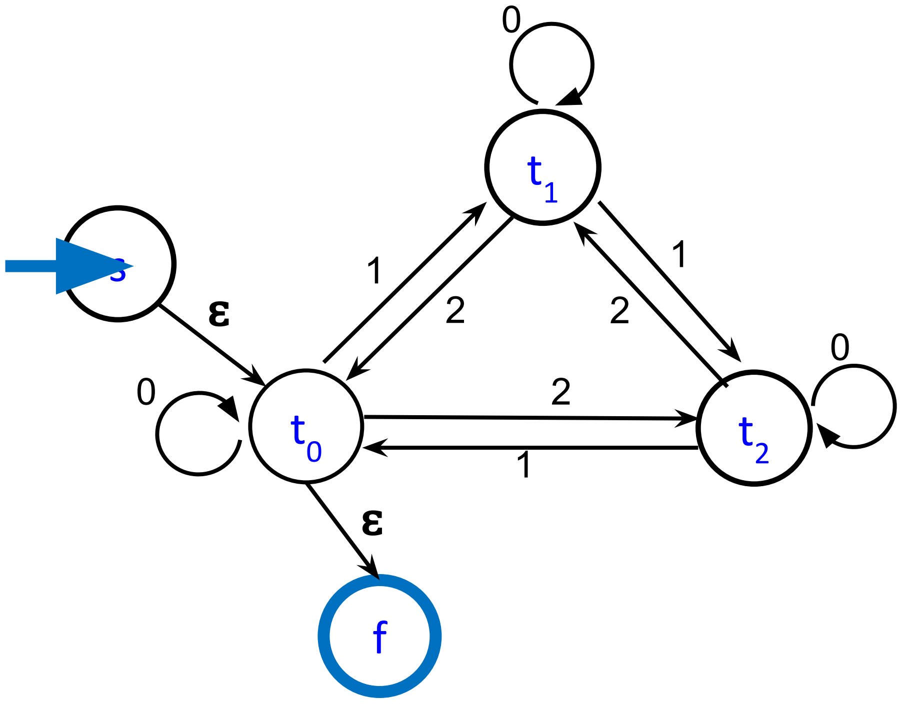
Consider the DFA for the mod 3 sum

- Accept strings from {0,1,2}* where the digits mod 3 sum of the digits is 0



Splicing out a state t_1

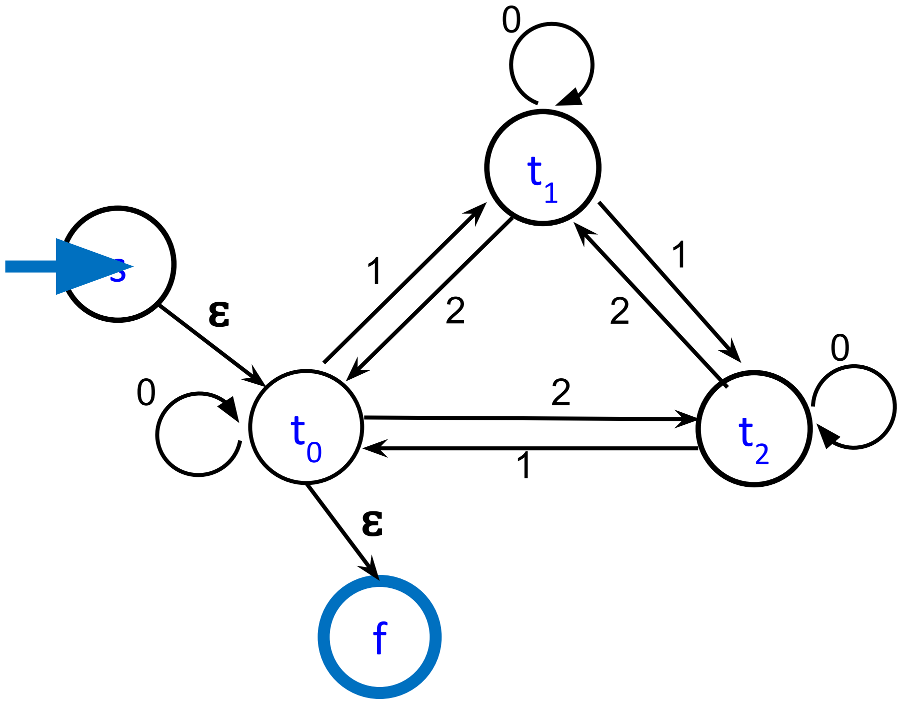
Create direct edges between neighbors of t_1 (so that we can delete it afterward)



Splicing out a state t_1

Regular expressions to add to edges

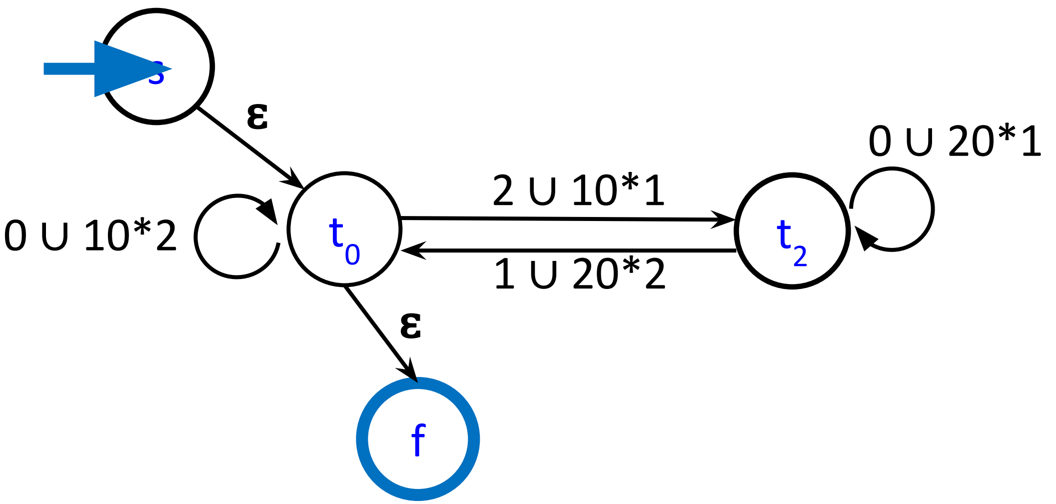
$t_0 \rightarrow t_1 \rightarrow t_0$: 10*2
$t_0 \rightarrow t_1 \rightarrow t_2$: 10*1
$t_2 \rightarrow t_1 \rightarrow t_0$: 20*2
$t_2 \rightarrow t_1 \rightarrow t_2$: 20*1



Splicing out a state t_1

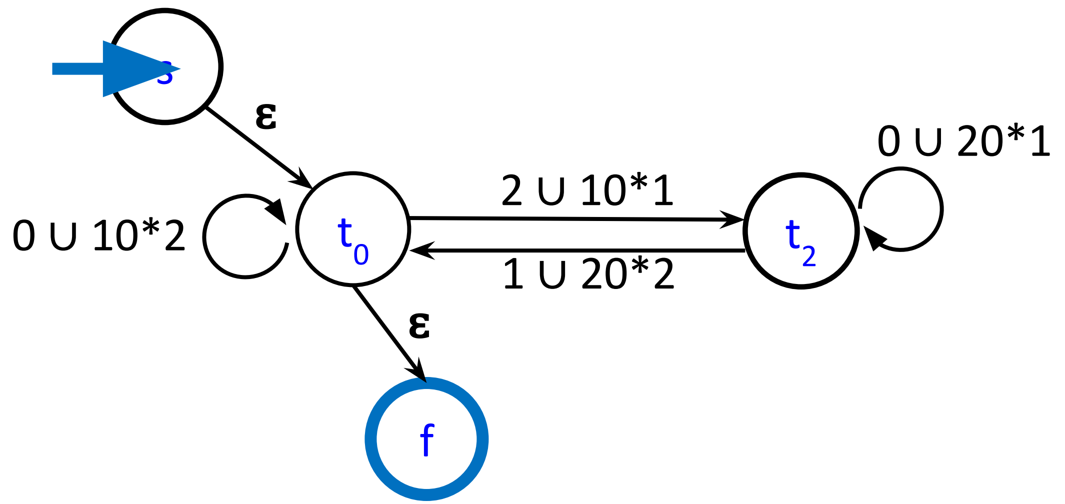
Delete t_1 now that it is redundant

$t_0 \longrightarrow t_1 \longrightarrow t_0$: 10^*2
$t_0 \longrightarrow t_1 \longrightarrow t_2$: 10^*1
$t_2 \longrightarrow t_1 \longrightarrow t_0$: 20^*2
$t_2 \longrightarrow t_1 \longrightarrow t_2$: 20^*1



Splicing out a state t_1

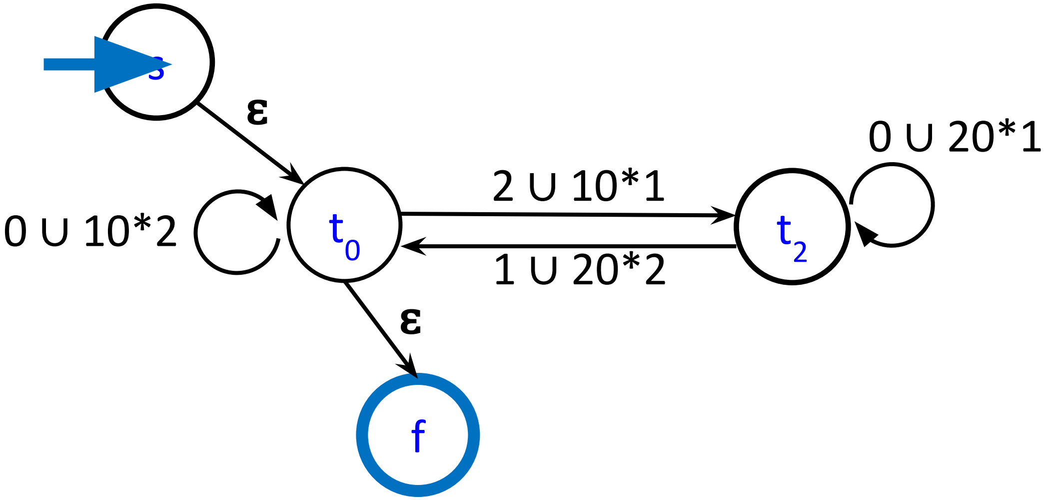
Create direct edges between neighbors of t_2
(so that we can delete it afterward)



Splicing out a state t_1

Regular expressions to add to edges

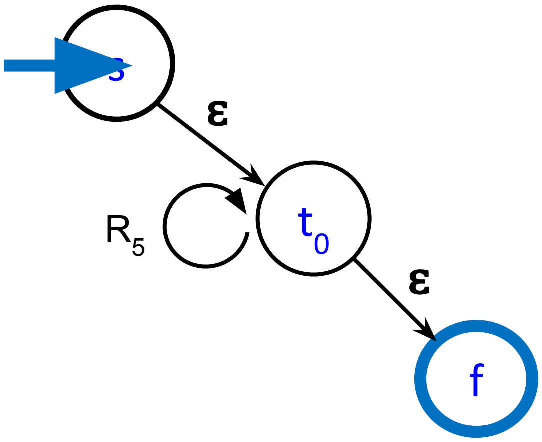
R_1:   0 ∪ 10*2
R_2:   2 ∪ 10*1
R_3:   1 ∪ 20*2
R_4:   0 ∪ 20*1



Splicing out state t_2 (and then t_0)

Delete t_2 now that it is redundant

R_1: $0 \cup 10^*2$
R_2: $2 \cup 10^*1$
R_3: $1 \cup 20^*2$
R_4: $0 \cup 20^*1$

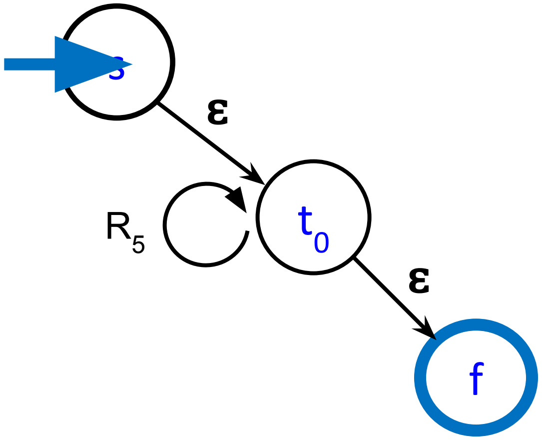


R_5: $R_1 \cup R_2R_4^*R_3$

Splicing out state t_2 (and then t_0)

Create direct (s,f) edge so we can delete t_0

R_1: $0 \cup 10^*2$
R_2: $2 \cup 10^*1$
R_3: $1 \cup 20^*2$
R_4: $0 \cup 20^*1$
R_5: $R_1 \cup R_2R_4{}^*R_3$

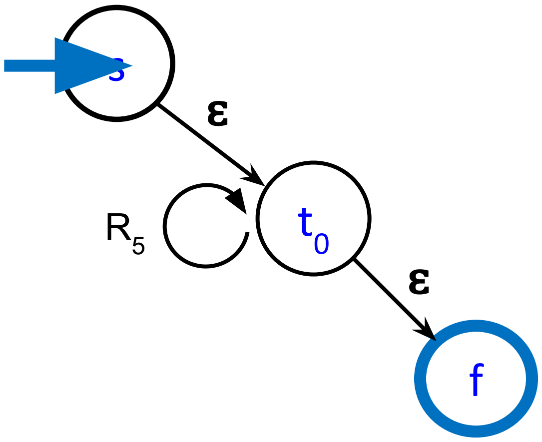


Splicing out state t_2 (and then t_0)

Regular expressions to add to edges

R_1: $0 \cup 10^*2$
R_2: $2 \cup 10^*1$
R_3: $1 \cup 20^*2$
R_4: $0 \cup 20^*1$
R_5: $R_1 \cup R_2R_4{}^*R_3$

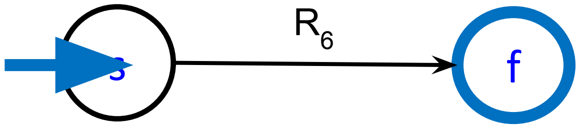
$t_0 \rightarrow t_1 \rightarrow t_0$: $R_5{}^*$



Splicing out state t_2 (and then t_0)

Delete t_0 now that it is redundant

R_1: $0 \cup 10^*2$
R_2: $2 \cup 10^*1$
R_3: $1 \cup 20^*2$
R_4: $0 \cup 20^*1$
R_5: $R_1 \cup R_2R_4{}^*R_3$

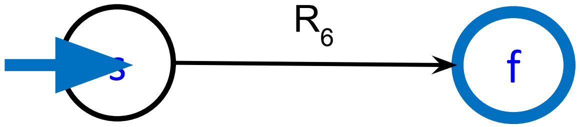


R_6: $R_5{}^*$

Splicing out state t_2 (and then t_0)

Regular expressions to add to edges

R_1: $0 \cup 10^*2$
R_2: $2 \cup 10^*1$
R_3: $1 \cup 20^*2$
R_4: $0 \cup 20^*1$
R_5: $R_1 \cup R_2R_4{}^*R_3$
R_6: $R_5{}^*$



Final regular expression: R_6 =
$(0 \cup 10^*2 \cup (2 \cup 10^*1)(0 \cup 20^*1)^*(1 \cup 20^*2))^*$

Application of FSMs: Pattern matching

- Given
  - a string s of n characters
  - a pattern p of m characters
  - usually $m \ll n$
- Find
  - all occurrences of the pattern p in the string s

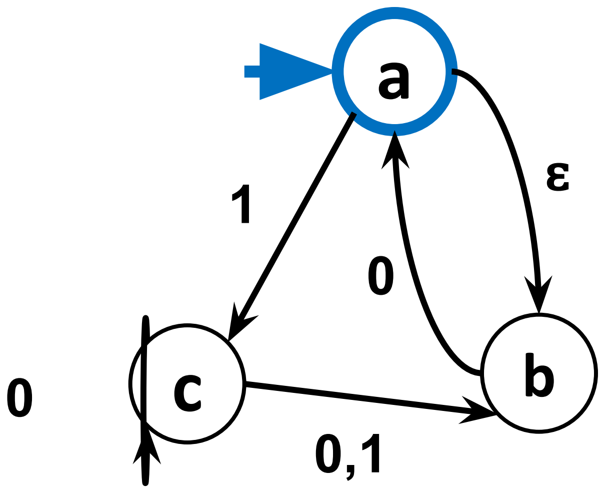
- Obvious algorithm:
  - try to see if p matches at each of the positions in s
    stop at a failed match and try matching at the next

Application of FSMs: Pattern Matching

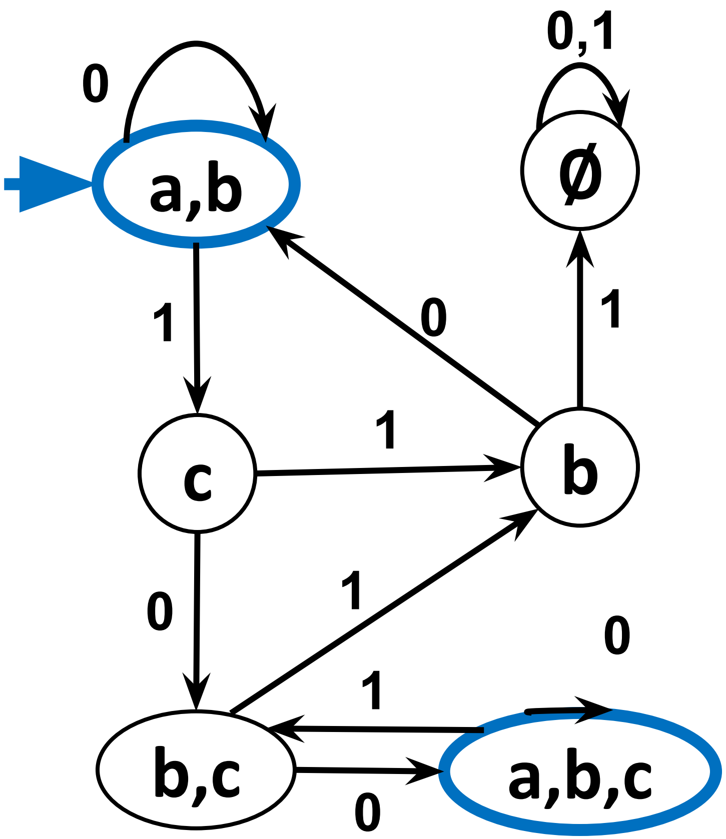
- **With DFAs can do this in $O(m + n)$ time.**

- **See Extra Credit problem on HW8 for some ideas of how to get to $O(m^2 + n)$.**

Last time: NFA to DFA



NFA



DFA

Exponential Blow-up in Simulating Nondeterminism

- **In general the DFA might need a state for every subset of states of the NFA**
  - **Power set of the set of states of the NFA**
  - n**-state NFA yields DFA with at most** 2^n **states**
  - **We saw an example where roughly** 2^n **is necessary**

    "Is the n^{th} char from the end a 1?"

The famous "P=NP?" question asks whether a similar blow-up is always necessary to get rid of nondeterminism for polynomial-time algorithms