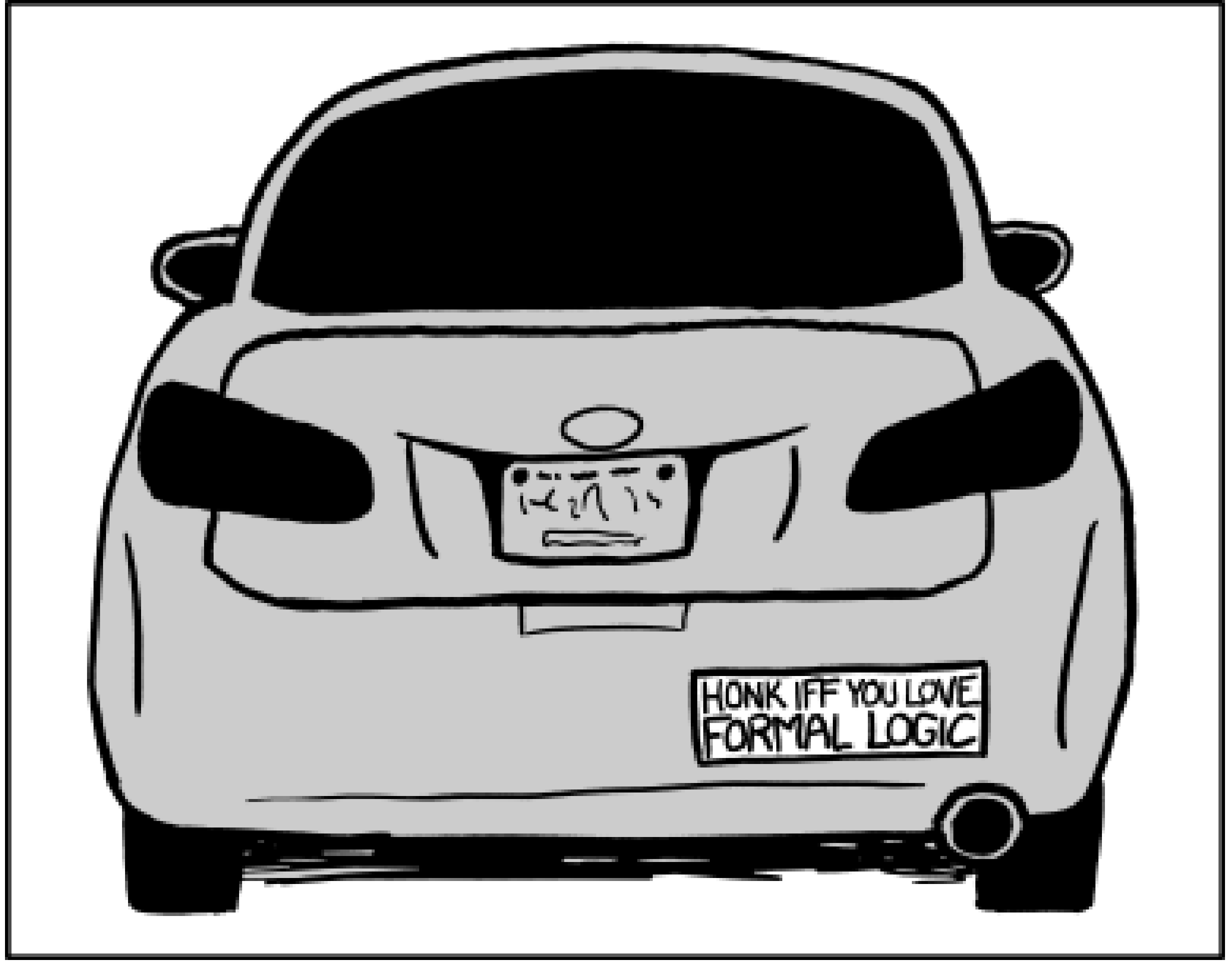

Propositional Logic

CSE 311: Foundations of Computing I
Lecture 1

Course Logistics

Assignment 0 is a syllabus quiz

Course Staff


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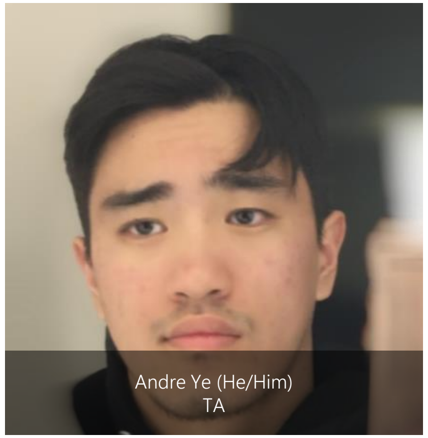
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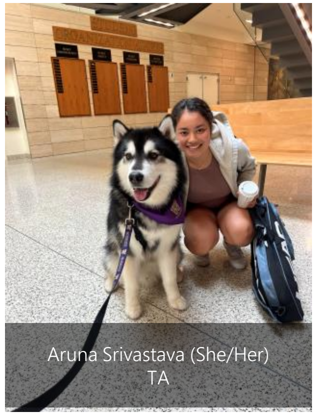
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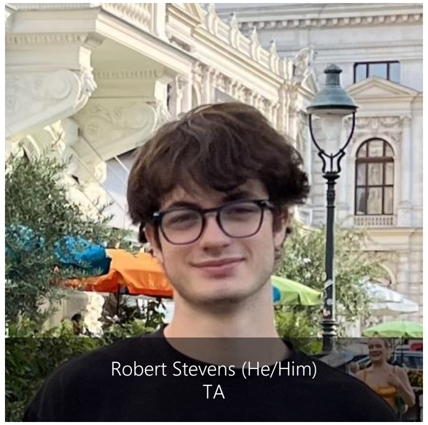
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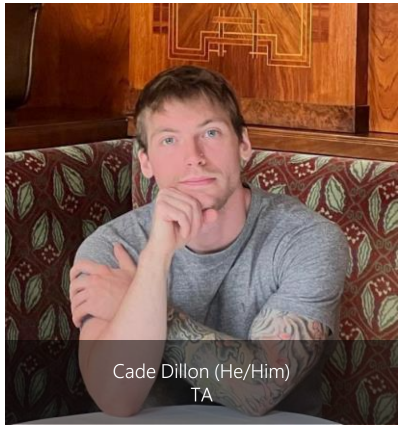
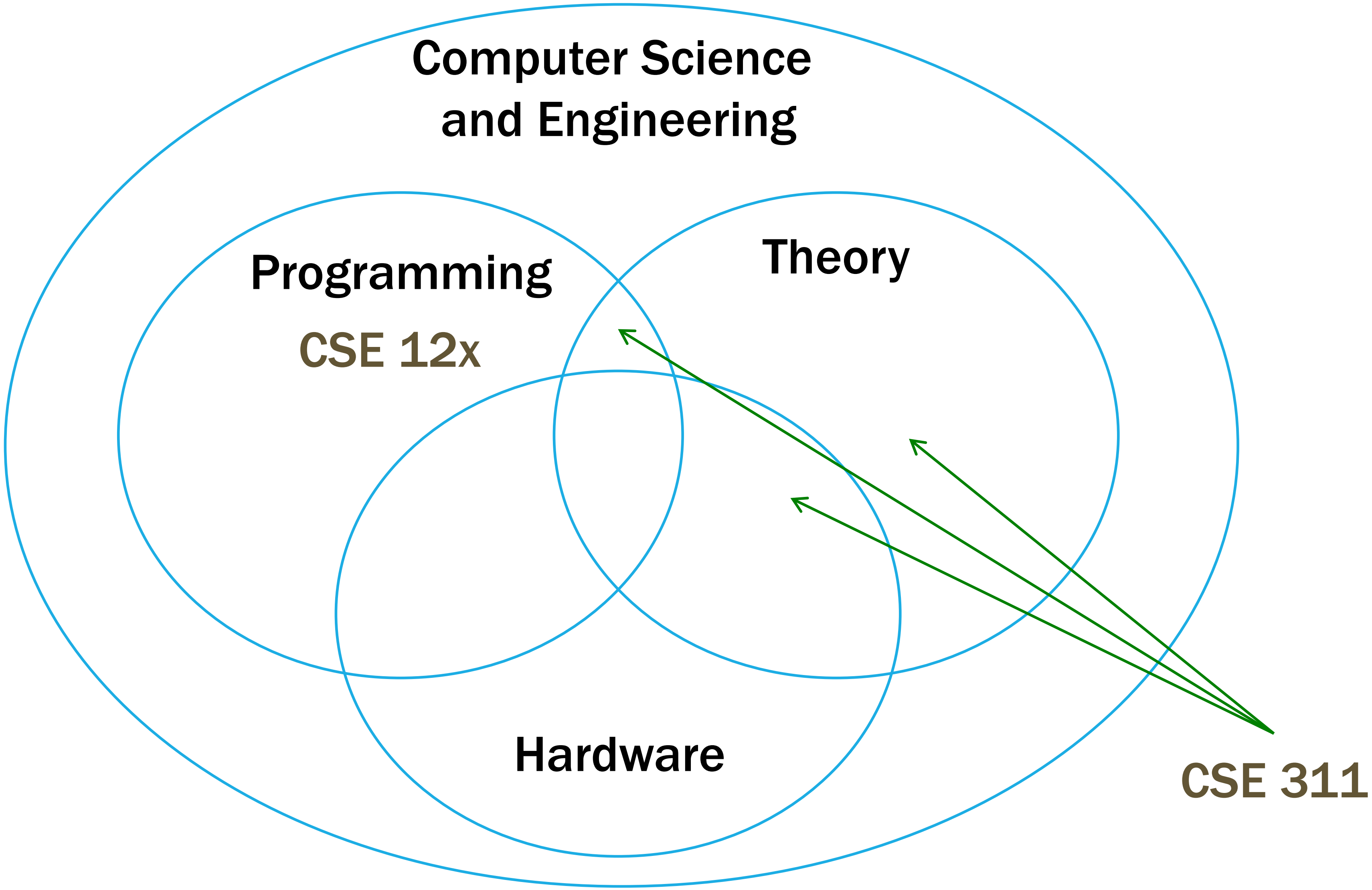

Cade Dillon (He/Him)
TA

Perspective



Using Algebra to Solve Problems

- It’s 1720 miles from Philadelphia to Denver. A train leaves Philly going 65 mph. Three hours later, a train leaves Denver at 40 mph. At what time do they collide?

- Let t be the time travelled by the Philly train
  - Philly train has traveled 65t miles
  - Denver train has travelled 40(t + 3) miles
  - Collide when 65t + 40(t + 3) = 1720
  - Solve for t

Course Goals

1. Learn to make & clearly communicate rigorous formal arguments
   - Mathematical Proofs
2. Understand mathematical objects that are widely used in CS
   - Number Theory, Set Theory, Recursively-Defined Functions
3. Explore and analyze models of computation
   - Regular Expressions, Context-Free Grammars
4. Develop a toolkit for approaching computational problems
   - Programmer → Computer Scientist

Lectures & Sections

- **Lectures**
  - Monday, Wednesday, Friday. Recorded.
- **Sections**
  - TA-led sections meet on Thursdays
  - Opportunity to practice and ask questions
  - Materials are posted, but sections aren’t recorded
  - Participation is part of the course grade
  - Can make-up absence from a couple of sections

Homework

- 8 written assignments
- Posted on Fridays, due the following Friday (with one exception)
  - HW 1 will release this Friday
  - For our course, we have a form you can fill out to request to turn in a homework late
  - We expect students to need 0-3 late days for the entire quarter
- Start Early and consider typesetting

Collaboration Policy

- Collaboration with others is **encouraged**
  - Do help other students learn
  - Do not help other students *avoid* learning
- Policy:
  - List all names of those you worked with
  - Don’t take away pictures or notes from discussions
  - Write up the final solutions on your own

Exams

- This course will have a midterm and final
  - The midterm will be during class on Monday July 22nd
  - The Final Exam will be from 3:30-5:30 on Friday August 16th in a room yet to be assigned
  - More information will be released closer to each exam time
- The exams are in person. If you cannot make the in person times, reach out as soon as possible to schedule an in person make up exam

Course Tools

- Course Website

(assignments, calendar)

Gradescope

(submissions, feedback)

Ed Discussion

(discussion board)

Canvas

(lecture recordings)

Propositional Logic

Why not use English?

English can be ambiguous or imprecise.

- Turn right here.

  Does “right” mean the direction, or “right now”?

- We saw her duck.

  Does “duck” mean the animal, or duck down?

- Buffalo buffalo Buffalo buffalo buffalo buffalo Buffalo buffalo.

  This means “Bison from Buffalo, that bison from Buffalo bully, themselves bully bison from Buffalo”

Benefits of formal logic

- We can state sentences precisely
- We can state sentences concisely
- The meaning of our sentences is unambiguous

Propositions: building blocks of logic

- Definition:

A proposition is a well-formed statement that is either true or false.

- Examples:
- All cats are mammals — True Proposition
- All mammals are cats — False Proposition

Analogy

Boolean (true / false) variables in Java

Are these Propositions?

- $2 + 2 = 5$
- Yes. Propositions can be false.
- $x + 2 = 5$
- No. Truth value depends on the value of x.
- $x + 2 = 3164$, where x is my PIN number
- Yes. This is either true or false.
- Akjsdf!
- No. This is gibberish.
- Who are you?
- No. This is a question, not a true or false statement.
- There is an infinite number of primes.
- Yes. We will prove this in a few weeks!

Proposition Notation

- We'll use variables to talk about arbitrary propositions
- Propositional variables: $p, q, r, s \ldots$
- Truth Values:

  $\mathbf{T}$ for true

  $\mathbf{F}$ for false

Atomic and Compound Propositions

- Definitions:

An atomic proposition is a proposition that can't be broken down any further.

- A compound proposition is a proposition that can be divided into simpler propositions.

- A logical connective combines atomic propositions into compound.

Compound Proposition Example

- It is raining in Seattle and it's June.

- Atomic Propositions:
- p: It is raining in Seattle
- q: It is June

Logical Connectives:

and ($\wedge$)

- Translation into Logic:
- $p \wedge q$

Analogy

Boolean variables p, q

Java and connective &&

All Logical Connectives in this class

Name	Logical Symbol	Java Symbol
Not	$\neg p$	$!\,p$
And	$p \wedge q$	$p\ \&\&\ q$
Or	$p \vee q$	$p\ \|\|\ q$
XOR	$p \oplus q$	$p\ \hat{}\ q$
Implication	$p \rightarrow q$	
Biconditional	$p \leftrightarrow q$	$p == q$

Not ($\neg$)

- $\neg p$ is true when p is false, and is false otherwise

- A **truth table** is a table of all possible truth values of an expression

p	$\neg p$
T	
F	

Not ($\neg$)

- $\neg p$ is true when p is false, and is false otherwise

- A **truth table** is a table of all possible truth values of an expression

p	$\neg p$
T	F
F	T

And (∧)

- $p \wedge q$ is true when p, q are both true, and is false otherwise

p	q	$p \wedge q$
T	T	
T	F	
F	T	
F	F	

And (∧)

- $p \wedge q$ is true when p, q are both true, and is false otherwise

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Or (∨)

- $p \vee q$ is true when **at least one** of p, q are true, and is false otherwise

p	q	$p \vee q$
T	T	
T	F	
F	T	
F	F	

Or (∨)

- $p \vee q$ is true when **at least one** of p, q are true, and is false otherwise

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Exclusive Or / XOR ($\oplus$)

- $p \oplus q$ is true when **exactly one** of p, q are true, and is false otherwise

p	q	$p \oplus q$
T	T	
T	F	
F	T	
F	F	

Exclusive Or / XOR ($\oplus$)

- $p \oplus q$ is true when **exactly one** of p, q are true, and is false otherwise

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Implication (→)

- $p \rightarrow q$ means "if p, then q"
- E.g. "If it is raining, then I have my umbrella"
- p: It is raining $p \rightarrow q$
- q: I have my umbrella

- Why is this a proposition? True or False statement

Implication

- $p \rightarrow q$ means "if p, then q"

- E.g. "If it is raining, then I have my umbrella"
- p: It is raining $p \rightarrow q$
- q: I have my umbrella

Think about this as did I lie?

	It is raining	It is not raining
I have my umbrella		
I do not have my umbrella		

Implication

- $p \rightarrow q$ means "if p, then q"

- E.g. "If it is raining, then I have my umbrella"
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- q: I have my umbrella

P	Q	P → Q
T	T	
T	F	
F	T	
F	F	

Think about this as did I lie?

	It is raining	It is not raining
I have my umbrella	No	No
I do not have my umbrella	Yes	No

Implication

- $p \rightarrow q$ means "if p, then q"
- E.g. "If it is raining, then I have my umbrella"
- p: It is raining

$$p \rightarrow q$$

- q: I have my umbrella

P	Q	P → Q
T	T	T
T	F	F
F	T	T
F	F	T

Think about this as did I lie?

	It is raining	It is not raining
I have my umbrella	No	No
I do not have my umbrella	Yes	No

Vacuous Truth

p: It is raining

q: I have my umbrella

$$p \rightarrow q$$

P	Q	P → Q
T	T	T
T	F	F
F	T	T
F	F	T

vacuous adjective

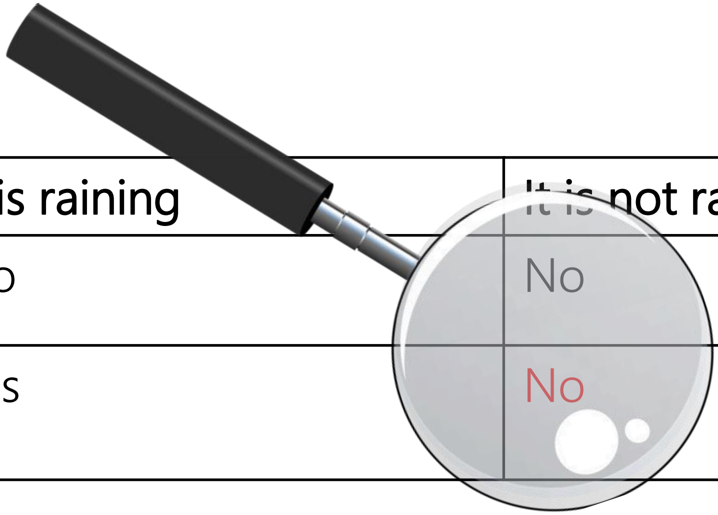
vac·u·ous ˈva-kyə-wəs

Synonyms of *vacuous* ›

1 : emptied of or lacking content

Vacuous Truth is when the premise of an implication is false

	It is raining	It is not raining
I have my umbrella	No	No
I do not have my umbrella	Yes	No



Translating Implications

Implication:

- p implies q
- whenever p is true q must be true
- if p then q
- q if p
- p is sufficient for q
- p only if q
- *q is necessary for p*

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Biconditional ($\leftrightarrow$), if and only if (iff)

- $p \leftrightarrow q$ is true when p, q have the same truth value, in other words are equal.

p	q	$\mathrm{p} \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Order of Operations

- $(\,)$ Parentheses
- $\neg$ Negation
- $\wedge$ And
- $\vee$ $\oplus$ Or, XOR
- $\rightarrow$ Implication
- $\leftrightarrow$ Biconditional

- Within the same order, evaluate from left to right.

- $p \vee \neg q \rightarrow r$ is the same as $\left(p \vee (\neg q)\right) \rightarrow r$.

Compound Proposition

Negation (not)	$\neg$
Conjunction (and)	$\wedge$
Disjunction (or)	$\vee$
Implication	$\rightarrow$
Biconditional	$\leftrightarrow$

"Unless I go to a café or to campus, I do not drink coffee, but also I don't go to cafés."

Compound Proposition

Negation (not)	$\neg$
Conjunction (and)	$\wedge$
Disjunction (or)	$\vee$
Implication	$\rightarrow$
Biconditional	$\leftrightarrow$

“Unless I go to a café or to campus, I do not drink coffee, but also I don’t go to cafés.”

P: I got to a cafe   q: I go to campus   r: I drink coffee

$$(\neg(p \vee q) \rightarrow \neg r) \wedge (\neg p)$$