Homework 8: Irregularity and Uncountability!! (aka DFAs/NFAs)

Due date: Wedensday, August 16th at 11:59 PM
If you want a late day, email jacob33@cs.washington.edu to request one.
If you work with others (and you should!), remember to follow the collaboration policy outlined in the syllabus.
In general, you are graded on your work's clarity and accuracy. Your solution should be clear enough that someone in the class who had not seen the problem before would understand it.
We sometimes describe approximately how long our explanations are. These are intended to help you understand approximately how much detail we expect. You can have longer explanations, but explanations significantly longer than necessary may receive deductions.

1. Just Irregular Guy [30 points]

Use the method described in lecture to prove that each of the following languages is **not regular**.

(a) All binary strings in the set $\{0^m 1^n : m, n \in \mathbb{N} \text{ and } m < n\}$.

(b) All strings of the form $x\#y$, with $x, y \in \{0,1\}^*$ and y a subsequence of x^R.

(Here x^R means the reverse of x. Also, a string w is a subsequence of another string z if you can delete some characters from z to arrive at w.)

2. Intergalactic Badminton [10 points]

The intergalactic olympics are on and Jeffrey must organize the badminton event for all of the infinite badminton players in the universe. Jeffrey needs to develop a protocol of assigning players to matches so that any two players in the universe will eventually play each other in a finite amount of time. Describe how Jeffrey can manage this, and create a matching such that this is possible (you may feel it helpful to draw a table and line).

3. Some closure at the end of the quarter [16 points]

We say that a property of languages is "closed" under an operation if applying the operation to languages with the property must produce another language with the property. For example, being regular is closed under union – for any regular languages L_1, L_2, the language $L_1 \cup L_2$ is also regular.

When taking the complement, let the universe be Σ^*.

(a) Prove that being regular is closed under complement, that is: for any regular language L, $\overline{L}$ is also regular. Recall that we have multiple equivalent definitions of regular, some are better suited to this problem than others! [7 points]

(b) Prove that Σ^* is a regular language. [2 points]

(c) Prove that it is not the case that being irregular (i.e., not regular) is closed under both complement and union.[1] For this problem, you should use proof by contradiction. (Hint: our proof uses the result of part b!) [7 points]

[1] That is show $\neg$(closed under complement $\wedge$ closed under union). If irregular languages were closed under complement and union, then the complement of every irregular language would be irregular and the union of any two irregular languages would be irregular.

4. Feedback [1 point]

Please keep track of how much time you spend on this homework and answer the following questions. This can help us calibrate future assignments and future iterations of the course, and can help you identify which areas are most challenging for you.

- How many hours did you spend working on this assignment (excluding any extra credit questions, if applicable)? Report your estimate to the nearest hour.
- Which problem did you spend the most time on?
- Any other feedback for us?