Quiz Section 8: CFGs, Relations, Graphs

Task 1 – Regular Expressions

a) Write a regular expression that matches base 10 numbers (e.g., there should be no leading zeroes).

b) Write a regular expression that matches all base-3 numbers that are divisible by 3.

c) Write a regular expression that matches all binary strings that contain the substring "111", but not the substring "000".

Task 2 – CFGs

Give CFGs for each of the following languages.

"Document" all the non-start variables in your grammar with an English description of the set of strings it generates. (You do not need to document the start variable because it is documented by the problem statement.)

a) All binary strings that start with 11.

b) All binary strings that contain at most one 1.

c) All binary strings that end in 00.

d) All binary strings that contain at least three 1's.

e) All strings over {0,1,2} with the same number of 1s and 0s and exactly one 2.

Hint: Try modifying the grammar from lecture for binary strings with the same number of 1s and 0s. (You may need to introduce new variables in the process.)

Task 3 – Good, Good, Good, Good Relations

Each part below defines a relation R on a set. For each part, first state whether R is reflexive, symmetric, antisymmetric, and/or transitive. Second, if a relation does *not* have a property, then state a counterexample. (If a relation *does* have a property, you don't need to do anything other than saying so.)

a) Let $R = \{(x, y) \ : \ x = y + 1\}$ on $\mathbb{N}$.

b) Let $R = \{(x, y) \ : \ x^2 = y^2\}$ on $\mathbb{R}$.

Task 4 – Relations

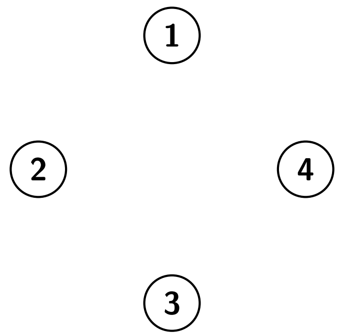
Let A be a set, and let R and S be relations on A. Suppose that R is reflexive.

a) Prove that $R \cup S$ is reflexive.

b) Prove that $R \subseteq R^2$. (Remember that R^2 is defined to be $R \circ R$.)

Task 5 – Closure

Draw the transitive-reflexive closure of $\{(1,2),(2,3),(3,4)\}$ as a directed graph. We have drawn the vertices for you.



Task 6 – String Relations

Let $\Sigma = \{0,1\}$. Define the relation R on Σ^* by $(x,y) \in R$ if and only if len(xy) is even. (Here xy is notation for the concatenation of the two strings x and y and len refers to the length of the string.)

Hint: In your proofs below, you may use the fact from lecture that len$(xy) =$ len$(x) +$ len(y).

a) Prove that R is reflexive.

b) Prove that R is symmetric.

c) Prove that R is transitive.

d) Is R antisymmetric? If so, prove it. If not, give a counterexample.