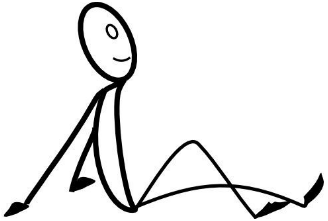
CSE 311 Section 06



Induction & Midterm Review

Announcements & Reminders

- HW4 grades released
  - Regrade requests will be open shortly
- HW5 due Tomorrow at 11:00pm
- Midterm (5/08) at regular class time
  - Lecture A: 10:30-11:20
  - Lecture B: 13:30-14:20
  - Attend your assigned lecture
- Midterm review
  - Monday, May 6th 5:00-8:00PM SIG 134
  - Bring questions!!!!!
- Book One-on-Ones on the course homepage!

(Weak) Induction Template

Let $P(n)$ be "(whatever you're trying to prove)".
We show $P(n)$ holds for all $n \in$ N by induction on n

Base Case: Show $P(b)$ is true.

Inductive Hypothesis: Suppose $P(k)$ holds for an arbitrary $k \geq b$.

Inductive Step: Show $P(k+1)$ (i.e. get $P(k) \rightarrow P(k+1)$)

Conclusion: Therefore, $P(n)$ holds for all n by the principle of induction.

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Let $P(n)$ be "(whatever you're trying to prove)".
We show $P(n)$ holds **for all n** $\in$ N by induction on n

Note: often you will condition n here, like "all natural numbers n" or "$n \geq 0$"

Base Case: Show $P(b)$ is true.

Inductive Hypothesis: Suppose $P(k)$ holds for an arbitrary $k \geq b$.

Inductive Step: Show $P(k+1)$ (i.e. get $P(k) \rightarrow P(k+1)$)

Conclusion: Therefore, $P(n)$ holds **for all n** by the principle of induction.

Match the earlier condition on n in your conclusion!

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Let $P(n)$ be "(whatever you're trying to prove)".
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P(n) IS A PREDICATE, IT HAS A BOOLEAN VALUE NOT A NUMERICAL ONE

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YOU MUST INTRODUCE AN ARBITRARY VARIABLE IN YOUR IH

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START WITH LHS OF K + 1 ONLY AND WORK TOWARD RHS

Conclusion: Therefore, $P(n)$ holds **for all n** by the principle of induction.

Weak Induction w/ Number Theory

Task 4b

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Then by the **Definition of Divides**, there exist integers j and k such that a=jc and b = kc

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Then by the **Definition of Divides**, there exist integers j and k such that a=jc and b = kc

Then a + b = jc + kc

Factoring out a constant we find, c(j + k)

Since **j+k is an integer** by definition we have c | (a + b)

Since a, b, and c were **arbitrary**, the claim holds.

Task 4b

ii. Show using induction that for any integer $n \geqslant 2$, given n numbers $a_1, a_2, \ldots, a_{n-1}, a_n$, for any integer c such that $c \mid a_i$ for $i = 1, 2, \ldots, n$, that

$$c \mid (a_1 + a_2 + \cdots + a_{n-1} + a_n).$$

In other words, if a number divides each term in a sum then that number divides the sum.

Task 4b

ii. Show using induction that for any integer $n \geqslant 2$, given n numbers $a_1, a_2, \ldots, a_{n-1}, a_n$, for any integer c such that $c \mid a_i$ for $i = 1, 2, \ldots, n$, that

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Let $P(n)$ be "given n numbers $a_1, a_2, \ldots, a_{n-1}, a_n$, for any integer c such that $c \mid a_i$ for $i = 1, 2, \ldots, n$, it holds that $c \mid (a_1 + a_2 + \cdots + a_n)$." We show $P(n)$ holds for all integer $n \geqslant 2$ by induction on n.

Conclusion: $P(n)$ holds for all integers $n \geqslant 2$ by induction the principle of induction.

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Base Case: $P(2)$ says that given two integers a_1 and a_2, for any integer c such that $c \mid a_1$ and $c \mid a_2$ it holds that $c \mid (a_1 + a_2)$. This is exactly part (a) so $P(2)$ holds.

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Inductive Hypothesis: Suppose that $P(k)$ holds for some arbitrary integer $k \geqslant 2$.

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Inductive Hypothesis: Suppose that $P(k)$ holds for some arbitrary integer $k \geqslant 2$.

Inductive Step: Let $a_1, a_2, \ldots, a_k, a_{k+1}$ be $k+1$ integers. Let c be arbitrary and suppose that $c \mid a_i$ for $i = 1, 2, \ldots, k+1$. Then we can write

$$a_1 + a_2 + \cdots + a_k + a_{k+1} = (a_1 + a_2 + \cdots + a_k) + a_{k+1}.$$

Conclusion: $P(n)$ holds for all integers $n \geqslant 2$ by induction the principle of induction.

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$$a_1 + a_2 + \cdots + a_k + a_{k+1} = (a_1 + a_2 + \cdots + a_k) + a_{k+1}.$$

The sum $a_1 + a_2 + \cdots + a_k$ has k terms and c divides all of them, meaning we can apply the inductive hypothesis. It says that $c \mid (a_1 + a_2 + \cdots + a_k)$. Since $c \mid (a_1 + a_2 + \cdots + a_k)$ and $c \mid a_{k+1}$, by part (a) we have,

$$c \mid (a_1 + a_2 + \cdots + a_k + a_{k+1}).$$

This shows $P(k+1)$.

Conclusion: $P(n)$ holds for all integers $n \geqslant 2$ by induction the principle of induction.

Strong Induction

Task 6

Consider the function $a(n)$ defined for $n \geqslant 1$ recursively as follows.

$$a(1) = 1$$

$$a(2) = 3$$

$$a(n) = 2a(n-1) - a(n-2) \text{ for } n \geqslant 3$$

Use strong induction to prove that $a(n) = 2n - 1$ for all $n \geqslant 1$.

Let $P(n)$ be "$a(n) = 2n - 1$". We will show that $P(n)$ is true for all $n \geqslant 1$ by strong induction.

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Base Cases $(n = 1, n = 2)$:
$(n = 1)$
$a(1) = 1 = 2 \cdot 1 - 1$

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So, $P(1)$ and $P(2)$ hold.

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Suppose that $P(j)$ is true for all integers $1 \leqslant j \leqslant k$ for some arbitrary $k \geqslant 2$.

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Suppose that $P(j)$ is true for all integers $1 \leqslant j \leqslant k$ for some arbitrary $k \geqslant 2$.

Inductive Step:
We will show $P(k+1)$ holds.

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Inductive Step:
We will show $P(k+1)$ holds.

$$a(k+1) = 2a(k) - a(k-1) \qquad \text{[Definition of } a\text{]}$$

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Inductive Step:
We will show $P(k+1)$ holds.

$$\begin{aligned} a(k+1) &= 2a(k) - a(k-1) && \text{[Definition of } a\text{]} \\ &= 2(2k-1) - (2(k-1) - 1) && \text{[Inductive Hypothesis]} \end{aligned}$$

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$$\begin{aligned}
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&= 2(2k-1) - (2(k-1) - 1) && \text{[Inductive Hypothesis]} \\
&= 2k + 1 && \text{[Algebra]} \\
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So, $P(k+1)$ holds.

Conclusion:
Therefore, $P(n)$ holds for all integers $n \geqslant 1$ by principle of strong induction.

Additional Weak Induction

Task 3

Define

$$H_i = \sum_{j=1}^{i} \frac{1}{j} = 1 + \frac{1}{2} + \cdots + \frac{1}{i}$$

The numbers H_i are called the *harmonic* numbers.

Prove that $H_{2^n} \geqslant 1 + \frac{n}{2}$ for all integers $n \geqslant 0$.

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The numbers H_i are called the *harmonic* numbers.
Prove that $H_{2^n} \geqslant 1 + \frac{n}{2}$ for all integers $n \geqslant 0$.

Let $P(n)$ be "$H_{2^n} \geqslant 1 + \frac{n}{2}$". We will prove $P(n)$ for all integers $n \geqslant 0$ by induction.

Base Case $(n = 0)$**:** $H_{2^0} = H_1 = \sum_{j=1}^{1} \frac{1}{j} = 1 \geqslant 1 + \frac{0}{2}$, so $P(0)$ holds.

Induction Hypothesis: Assume that $H_{2^k} \geqslant 1 + \frac{k}{2}$ for some arbitrary integer $k \geqslant 0$.

Induction Step: Goal: Show $H_{2^{k+1}} \geqslant 1 + \frac{k+1}{2}$

$$\begin{aligned}
H_{2^{k+1}} &= \sum_{j=1}^{2^{k+1}} \frac{1}{j} \\
&= \sum_{j=1}^{2^k} \frac{1}{j} + \sum_{j=2^k+1}^{2^{k+1}} \frac{1}{j} \\
&\geqslant 1 + \frac{k}{2} + \sum_{j=2^k+1}^{2^{k+1}} \frac{1}{j} && \text{[Induction Hypothesis]} \\
&\geqslant 1 + \frac{k}{2} + 2^k \cdot \frac{1}{2^{k+1}} && \text{[There are } 2^k \text{ terms in } [2^k+1, 2^{k+1}] \text{ and each is at least } \tfrac{1}{2^{k+1}}\text{]} \\
&\geqslant 1 + \frac{k}{2} + \frac{2^k}{2^{k+1}} \\
&\geqslant 1 + \frac{k}{2} + \frac{1}{2} \geqslant 1 + \frac{k+1}{2}
\end{aligned}$$

So $P(k+1)$ follows.

Conclusion: $P(n)$ holds for all integers $n \geqslant 0$ by induction.

Midterm Review: Translation

Problem 1 – Translation

Let your domain of discourse be all coffee drinks. You should use the following predicates:

- soy(x) is true iff x contains soy milk.
- whole(x) is true iff x contains whole milk.
- sugar(x) is true iff x contains sugar
- decaf(x) is true iff x is not caffeinated.
- vegan(x) is true iff x is vegan.
- RobbieLikes(x) is true iff Robbie likes the drink x.

Translate each of the following statements into predicate logic. You may use quantifiers, the predicates above, and usual math connectors like = and ≠.

a) Coffee drinks with whole milk are not vegan

b) Robbie only likes one coffee drink, and that drink is not vegan

c) There is a drink that has both sugar and soy milk.

Work on this problem with the people around you.

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- $\text{decaf}(x)$ is true iff x is not caffeinate
- $\text{vegan}(x)$ is true iff x is vegan
- $\text{RobbieLikes}(x)$ is true iff Robbie likes the drink x

a) Coffee drinks with whole milk are not vegan

$$\forall x(\text{whole}(x) \rightarrow \neg\text{vegan}(x))$$

a) Robbie only likes one coffee drink, and that drink is not vegan

a) There is a drink that has both sugar and soy milk.

Problem 1 – Translation

- soy(x) is true iff x contains soy milk
- whole(x) is true iff x contains whole milk
- sugar(x) is true iff x contains sugar
- decaf(x) is true iff x is not caffeinate
- vegan(x) is true iff x is vegan
- RobbieLikes(x) is true iff Robbie likes the drink x

a) Coffee drinks with whole milk are not vegan

$$\forall x(\text{whole}(x) \rightarrow \neg\text{vegan}(x))$$

a) Robbie only likes one coffee drink, and that drink is not vegan

$$\exists x \forall y(\text{RobbieLikes}(x) \wedge \neg\,\text{Vegan}(x) \wedge [\text{RobbieLikes}(y) \rightarrow x = y])$$

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$$\exists x \forall y(\text{RobbieLikes}(x) \land \neg \text{Vegan}(x) \land [\text{RobbieLikes}(y) \rightarrow x = y])$$

Or $\exists x(\text{RobbieLikes}(x) \land \neg \text{Vegan}(x) \land \forall y[\text{RobbieLikes}(y) \rightarrow x = y])$

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a) There is a drink that has both sugar and soy milk.

$$\exists x\big(\text{sugar}(x) \wedge \text{soy}(x)\big)$$

Problem 1 – Translation

Let your domain of discourse be all coffee drinks. You should use the following predicates:

- soy(x) is true iff x contains soy milk.
- whole(x) is true iff x contains whole milk.
- sugar(x) is true iff x contains sugar
- decaf(x) is true iff x is not caffeinated.
- vegan(x) is true iff x is vegan.
- RobbieLikes(x) is true iff Robbie likes the drink x.

Translate the following symbolic logic statement into a (natural) English sentence. Take advantage of domain restriction.

$$\forall x([\text{decaf}(x) \land \text{RobbieLikes}(x)] \rightarrow \text{sugar}(x))$$

Work on this problem with the people around you.

Problem 1 – Translation

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$$\forall x([\text{decaf}(x) \wedge \text{RobbieLikes}(x)] \rightarrow \text{sugar}(x))$$

Every decaf drink that Robbie likes has sugar.

Problem 1 – Translation

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- RobbieLikes(x) is true iff Robbie likes the drink x

$$\forall x([\text{decaf}(x) \wedge \text{RobbieLikes}(x)] \rightarrow \text{sugar}(x))$$

Every decaf drink that Robbie likes has sugar.

Statements like "For every decaf drink, if Robbie likes it then it has sugar" are equivalent, but only partially take advantage of domain restriction.

That’s All, Folks!

Thanks for coming to section this week!
Any questions?