CSE 311 Section 2

Logic and Equivalences

Announcements & Reminders

- Sections are Graded
  - You will be graded on section participation, so please try to come ☺

- HW1 due yesterday on Gradescope
  - 1 late day per HW (3 late days total); just submit late to use late day
- HW 2 released today, due next Wednesday @ 11 PM
- Check the course website for OH times!
  - There are office hours every day, so come visit if you have questions!
- 1 on 1 office hours will start soon: More info to come

References

- Helpful reference sheets can be found on the course website!
  - https://courses.cs.washington.edu/courses/cse311/23au/resources/
- How to LaTeX (found on Assignments page of website):
  - https://courses.cs.washington.edu/courses/cse311/23au/assignments/HowToLaTeX.pdf
- Equivalence Reference Sheet
  - https://courses.cs.washington.edu/courses/cse311/23au/resources/reference-logical_equiv.pdf
  - https://courses.cs.washington.edu/courses/cse311/23au/resources/logicalConnectPoster.pdf
- Boolean Algebra Reference Sheet
  - https://courses.cs.washington.edu/courses/cse311/23au/resources/reference-boolean-alg.pdf
- Plus more!

Tautology if it is always true
Contradiction if it is always false
Contingency if it can be either true or false

Symbolic Proofs with Cozy

Navigate to: https://cozy.cs.washington.edu/service/home

Problem 1 – Equivalences

a) p → ¬p ∧ ¬p→ p vs. F: **tinyurl.com/CSE311S2**

Remember these identities!

These identities hold for all propositions p, q, r

- Identity
  - $p \wedge \mathrm{T} \equiv p$
  - $p \vee \mathrm{F} \equiv p$
- Domination
  - $p \vee \mathrm{T} \equiv \mathrm{T}$
  - $p \wedge \mathrm{F} \equiv \mathrm{F}$
- Idempotent
  - $p \vee p \equiv p$
  - $p \wedge p \equiv p$
- Commutative
  - $p \vee q \equiv q \vee p$
  - $p \wedge q \equiv q \wedge p$
- Associative
  - $(p \vee q) \vee r \equiv p \vee (q \vee r)$
  - $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
- Distributive
  - $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
  - $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
- Absorption
  - $p \vee (p \wedge q) \equiv p$
  - $p \wedge (p \vee q) \equiv p$
- Negation
  - $p \vee \neg p \equiv \mathrm{T}$
  - $p \wedge \neg p \equiv \mathrm{F}$
- DeMorgan's Laws
  - $\neg(p \vee q) \equiv \neg p \wedge \neg q$
  - $\neg(p \wedge q) \equiv \neg p \vee \neg q$
- Double Negation
  - $\neg\neg p \equiv p$
- Law of Implication
  - $p \to q \equiv \neg p \vee q$
- Contrapositive
  - $p \to q \equiv \neg q \to \neg p$

Problem 1 – Equivalences

a) $p \to \neg p \land \neg p \to p$ vs. F

$(p \to \neg p) \land (\neg p \to p)$	$\equiv$	$(\neg p \lor \neg p) \land (\neg\neg p \lor p)$	Law of Implication
	$\equiv$	$(\neg p \lor \neg p) \land (p \lor p)$	Double Negation
	$\equiv$	$\neg p \land p$	Idempotence
	$\equiv$	$p \land \neg p$	Commutativity
	$\equiv$	F	Negation

Problem 1 – Equivalences

b) $\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r)$

The link: **tinyurl.com/CSE311S2b**

Remember these identities!

These identities hold for all propositions p, q, r

- Identity
  - $p \wedge \mathrm{T} \equiv p$
  - $p \vee \mathrm{F} \equiv p$
- Domination
  - $p \vee \mathrm{T} \equiv \mathrm{T}$
  - $p \wedge \mathrm{F} \equiv \mathrm{F}$
- Idempotent
  - $p \vee p \equiv p$
  - $p \wedge p \equiv p$
- Commutative
  - $p \vee q \equiv q \vee p$
  - $p \wedge q \equiv q \wedge p$
- Associative
  - $(p \vee q) \vee r \equiv p \vee (q \vee r)$
  - $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
- Distributive
  - $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
  - $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
- Absorption
  - $p \vee (p \wedge q) \equiv p$
  - $p \wedge (p \vee q) \equiv p$
- Negation
  - $p \vee \neg p \equiv \mathrm{T}$
  - $p \wedge \neg p \equiv \mathrm{F}$
- DeMorgan's Laws
  - $\neg(p \vee q) \equiv \neg p \wedge \neg q$
  - $\neg(p \wedge q) \equiv \neg p \vee \neg q$
- Double Negation
  - $\neg\neg p \equiv p$
- Law of Implication
  - $p \rightarrow q \equiv \neg p \vee q$
- Contrapositive
  - $p \rightarrow q \equiv \neg q \rightarrow \neg p$

Problem 1 – Equivalences

b) $\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r)$

(i)

p	r	s	$\neg p$	$(s \rightarrow r)$	$(p \vee r)$	$\neg p \rightarrow (s \rightarrow r)$	$s \rightarrow (p \vee r)$
T	T	T	F	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	F	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	F	T	T	F	F	F	F
F	F	F	T	T	F	T	T

(ii)

$\neg p \rightarrow (s \rightarrow r)$	$\equiv$	$\neg\neg p \vee (s \rightarrow r)$	Law of Implication
	$\equiv$	$p \vee (s \rightarrow r)$	Double Negation
	$\equiv$	$p \vee (\neg s \vee r)$	Law of Implication
	$\equiv$	$(p \vee \neg s) \vee r$	Associativity
	$\equiv$	$(\neg s \vee p) \vee r$	Commutativity
	$\equiv$	$\neg s \vee (p \vee r)$	Associativity
	$\equiv$	$s \rightarrow (p \vee r)$	Law of Implication

Problem 5

Problem 5 – Canonical Forms

Consider the functions $F(A,B,C)$ and $G(A,B,C)$ specified by the following truth table:

a) Write the DNF and CNF expressions for $F(A,B,C)$.

b) Write the DNF and CNF expressions for $G(A,B,C)$.

A	B	C	F(A,B,C)	G(A,B,C)
T	T	T	T	F
T	T	F	T	T
T	F	T	F	F
T	F	F	F	F
F	T	T	T	T
F	T	F	T	F
F	F	T	F	T
F	F	F	T	F

Problem 5 – Canonical Forms

b) Write the **CNF** expressions for G(A,B,C)

A	*B*	*C*	*G(A,B,C)*
T	T	T	F
T	T	F	T
T	F	T	F
T	F	F	F
F	T	T	T
F	T	F	F
F	F	T	T
F	F	F	F

Work on part (b) with the people around you, and then we'll go over it together!

Problem 5 – Canonical Forms

CNF: (AND of ORs)

	A	B	C	G(A,B,C)
(a'+b'+c')	T	T	T	F
	T	T	F	T
(a'+b+c')	T	F	T	F
(a'+b+c)	T	F	F	F
	F	T	T	T
(a+b'+c)	F	T	F	F
	F	F	T	T
(a+b+c)	F	F	F	F

(a'+b'+c')(a'+b+c')(a'+b+c)(a+b'+c)(a+b+c)

Problem 5 – Canonical Forms

(c) Simplify the CNF form for G(A,B,C) that you found in (b)

Remember these identities!

Null

$$X + 1 = 1$$
$$X \bullet 0 = 0$$

Idempotency

$$X + X = X$$
$$X \bullet X = X$$

Involution

$$(X')' = X$$

Uniting

$$X \bullet Y + X \bullet Y' = X$$
$$(X + Y) \bullet (X + Y') = X$$

Absorbtion

$$X + X \bullet Y = X$$
$$(X + Y') \bullet Y = X \bullet Y$$
$$X \bullet (X + Y) = X$$
$$(X \bullet Y') + Y = X + Y$$

DeMorgan

$$(X + Y + \cdots)' = X' \bullet Y' \bullet \cdots$$
$$(X \bullet Y \bullet \cdots)' = X' + Y' + \cdots$$

Problem 5 – Canonical Forms

(c) Simplify the CNF form for G(A,B,C) that you found in (b)

CNF from (b):

$(a'+b'+c')(a'+b+c')(a'+b+c)(a+b'+c)(a+b+c)$

Problem 5 – Canonical Forms

(c) Simplify the CNF form for G(A,B,C) that you found in (b)

CNF from (b):

$(a'+b'+c')(a'+b+c')(a'+b+c)(a+b'+c)(a+b+c)$

$\equiv (a' + c')(a' + b + c)(a + b' + c)(a + b + c)$ Uniting

Problem 5 – Canonical Forms

(c) Simplify the CNF form for G(A,B,C) that you found in (b)

CNF from (b):

$(a'+b'+c')(a'+b+c')(a'+b+c)(a+b'+c)(a+b+c)$

$\equiv (a' + c')(a' + b + c)(a + b' + c)(a + b + c)$ Uniting

$\equiv (a' + c')(a' + b + c)(a + b' + c)(a + b + c)\ (a + b + c)$ Idempotency

Problem 5 – Canonical Forms

(c) Simplify the CNF form for G(A,B,C) that you found in (b)

CNF from (b):

$(a'+b'+c')(a'+b+c')(a'+b+c)(a+b'+c')(a+b+c)$	
$\equiv (a' + c')(a' + b + c)(a + b' + c)(a + b + c)$	Uniting
$\equiv (a' + c')(a' + b + c)(a + b' + c)(a + b + c)\ (a + b + c)$	Idempotency
$\equiv (a' + c')\ (b + c)\ (a + c)$	Uniting

Problem 5 – Canonical Forms

(c) Simplify the CNF form for G(A,B,C) that you found in (b)

CNF from (b):

$(a'+b'+c')(a'+b+c')(a'+b+c)(a+b'+c')(a+b+c)$	
$\equiv (a' + c')(a' + b + c)(a + b' + c)(a + b + c)$	Uniting
$\equiv (a' + c')(a' + b + c)(a + b' + c)(a + b + c)\ (a + b + c)$	Idempotency
$\equiv (a' + c')\ (b + c)\ (a + c)$	Uniting
$\equiv (a' + c')(ab + c)$	Distributivity

Problem 2

Problem 2 - Non-Equivalence

a) $p \rightarrow r$ vs. $r \rightarrow p$

Problem 2 - Non-Equivalence

a) $p \to r$ vs. $r \to p$

p	r	$p \to r$	$r \to p$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

Problem 2 - Non-Equivalence

a) $p \to r$ vs. $r \to p$

p	r	$p \to r$	$r \to p$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

They differ when P is true and R is false as the first is vacuously true, while the other is false

That’s All, Folks!

Thanks for coming to section this week!
Any questions?