CSE 311 Section 1

Propositional Logic

Announcements & Reminders

- Sections are Graded
  - You will be graded on section participation, so please try to come ☺

- Section Materials
  - Handouts will be provided in at each section
  - Worksheets and sample solutions will be available on the course calendar later this evening

Your TAs

- TA 1
  - content

- TA 2
  - content

Tips for 311!

- Tackling challenging homework problems may feel intimidating at first but **don't go at it alone**! Find study groups, join us in office hours, book one-on-ones, and ask questions on Ed.

- Section will often be challenging and fast but valuable for your learning. **This is your time to ask lots of questions and clarify your learning**!
- Sometimes homework problems will mirror section problems, use that to your advantage!

- This class is the best time to learn how to Latex, please consider learning now as it will save you time for future courses! Feel free to come to office hours to get help with Latex!

- We have created an example latex template that you can find here: https://www.overleaf.com/read/rnxmrjvgbtrx#e59d4e

Icebreaker

- Small groups of 4-6ish
- Please share with your group
  - Your name
  - Number of years in department/ at UW
  - What was something fun you did over Winter break?
  - What are you concerned about for 311 / what are you excited about?
- Then, share how you like to eat your potatoes (baked, fried, chips, etc.)
- We’ll go around and see what style of potato is most popular!

Propositions & Implications

Quick Concept Review

- **Propositions** are statements with a boolean truth value!
  - "**The AQI of Seattle is 50**" is a proposition. We know it's either true or false.
  - "**The AQI of Seattle?**" is not. Suddenly it could be hundreds of values.
  - In formal logic, we like to assign a proposition into a variable for later use.
- **Logical connectives** connect propositions to form new propositions!

$$\neg p$$

$$p \wedge q$$

$$p \vee q$$

$$p \rightarrow q$$

$$p \leftrightarrow q$$

Truth Tables

Gives us a simple way to describe how logical connectives operate

p	¬p
T	F
F	T

p	q	p∧q
T	T	T
T	F	F
F	T	F
F	F	F

p	q	p∨q
T	T	T
T	F	T
F	T	T
F	F	F

Implications

Some common formulations:

- p implies q
- whenever p is true q must be true
- If p then q
- q if p
- p is sufficient for q
- p only if q
- q is necessary for p

p	q	p → q
T	T	T
T	F	F
F	T	T
F	F	T

Vacuous truths: a false hypothesis, but true truth value

“Only if”

I attended my 8:30am class only if I woke up early

Which is equivalent?

If I woke up early then I attended my 8:30 am class

or

If I attended my 8:30 am class then I woke up early

“Only if”

I attended my 8:30am class only if I woke up early

If I woke up early then I attended my 8:30 am class

NOT Equivalent: The original statement **does not specify** what happens **when you wake up early**, you can wake up early to go play tennis in the morning!

If I attended my 8:30 am class then I woke up early

Equivalent: The original statement only **specifies exactly** what happened **when you went to your 8:30 class**, you **must** have woken up early. Nothing else could have happened for you to be attending the 8:30 class.

Problem 1 – Warm Up

Steps:

1. Create propositional variables
2. Replace all propositions with created variables
3. Replace the operators

(a) If I am lifting weights this afternoon, then I do a warm-up exercise.

(b) If I am cold and going to bed or I am two-years old, then I carry a blanket.

Problem 1a – Warm Up

Steps:

1. Create propositional variables
2. Replace all propositions with created variables
3. Replace the operators

a) If I am lifting weights this afternoon, then I do a warm-up exercise.

Problem 1a – Warm Up

Steps:

1. Create propositional variables
2. Replace all propositions with created variables
3. Replace the operators

a) If I am lifting weights this afternoon, then I do a warm-up exercise.

Step 1

p: I am lifting weights this afternoon

q: I do a warm-up exercise

Problem 1a – Warm Up

Steps:

1. Create propositional variables
2. Replace all propositions with created variables
3. Replace the operators

a) If I am lifting weights this afternoon, then I do a warm-up exercise.

Step 1

p: I am lifting weights this afternoon

q: I do a warm-up exercise

Step 2

If p **then** q

Problem 1a – Warm Up

Steps:

1. Create propositional variables
2. Replace all propositions with created variables
3. Replace the operators

a) If I am lifting weights this afternoon, then I do a warm-up exercise.

Step 1

p: I am lifting weights this afternoon

q: I do a warm-up exercise

Step 2

If p **then** q

Step 3

$p \to q$

Problem 2

Problem 2 – If I can translate, then...

1. Create propositional variables
2. Replace all propositions with created variables
3. Replace the operators

a) Whenever I walk my dog, I make new friends.

b) I will drink coffee, if Starbucks is open or my coffeemaker works.

c) Being a U.S. citizen and over 18 is sufficient to be eligible to vote.

d) I can go home only if I have finished my homework.

e) Having an internet connection is necessary to log onto zoom.

f) I am a student because I attend university.

Work on parts (a), (c), and (f) with the people around you, and then we'll go over it together!

Problem 2 – If I can translate, then…

a) Whenever I walk my dog, I make new friends.

1. Create propositional variables
2. Replace all propositions with created variables
3. Replace the operators

Problem 2 – If I can translate, then...

1. Create propositional variables
2. Replace all propositions with created variables
3. Replace the operators

a) Whenever I walk my dog, I make new friends.

Step 1
p: I walk my dog
q: I make new friends

Problem 2 – If I can translate, then…

1. Create propositional variables
2. Replace all propositions with created variables
3. Replace the operators

a) Whenever I walk my dog, I make new friends.

Step 1

p: I walk my dog

q: I make new friends

Step 2

Whenever p, q

If p then q

Problem 2 – If I can translate, then…

1. Create propositional variables
2. Replace all propositions with created variables
3. Replace the operators

a) Whenever I walk my dog, I make new friends.

Step 1
p: I walk my dog
q: I make new friends

Step 2
Whenever p, q
If p then q

Step 3
$p \to q$

Problem 2 – If I can translate, then…

c) Being a U.S. citizen and over 18 is sufficient to be eligible to vote.

1. Create propositional variables
2. Replace all propositions with created variables
3. Replace the operators

Problem 2 – If I can translate, then…

1. Create propositional variables
2. Replace all propositions with created variables
3. Replace the operators

c) Being a U.S. citizen and over 18 is sufficient to be eligible to vote.

Step 1

p: One is a U.S. Citizen

q: One is over 18

r: One is eligible to vote

Problem 2 – If I can translate, then...

1. Create propositional variables
2. Replace all propositions with created variables
3. Replace the operators

c) Being a U.S. citizen and over 18 is sufficient to be eligible to vote.

Step 1

p: One is a U.S. Citizen
q: One is over 18
r: One is eligible to vote

Step 2

Being p and q is sufficient for r
If p and q then r

Problem 2 – If I can translate, then...

1. Create propositional variables
2. Replace all propositions with created variables
3. Replace the operators

c) Being a U.S. citizen and over 18 is sufficient to be eligible to vote.

Step 1

p: One is a U.S. Citizen

q: One is over 18

r: One is eligible to vote

Step 2

Being p and q is sufficient for r

If p and q then r

Step 3

$(p \wedge q) \rightarrow r$

Problem 1b

Problem 1b

Steps:

1. Create propositional variables
2. Replace all propositions with created variables
3. Replace the operators

b) If I am cold and going to bed or I am two-years old, then I carry a blanket.

Work on this problem with the people around you, and then we'll go over it together!

Problem 1b

Steps:

1. Create propositional variables
2. Replace all propositions with created variables
3. Replace the operators

b) If I am cold and going to bed or I am two-years old, then I carry a blanket.

Problem 1b

Steps:

1. Create propositional variables
2. Replace all propositions with created variables
3. Replace the operators

b) If I am cold and going to bed or I am two-years old, then I carry a blanket.

Step 1

p: I am cold

q: I am going to bed

r: I am two-years old

s: I carry a blanket

NOTE: you need a subject for each proposition. “Going to bed” is not a proper proposition, you need to add the “I am” to make it a valid sentence, and thus a valid proposition!!!

Problem 1b

Steps:

1. Create propositional variables
2. Replace all propositions with created variables
3. Replace the operators

b) If I am cold and going to bed or I am two-years old, then I carry a blanket.

Step 1

p: I am cold
q: I am going to bed
r: I am two-years old
s: I carry a blanket

Step 2

If p and q or r, then s

Problem 1b

Steps:

1. Create propositional variables
2. Replace all propositions with created variables
3. Replace the operators

b) If I am cold and going to bed or I am two-years old, then I carry a blanket.

Step 1

p: I am cold

q: I am going to bed

r: I am two-years old

s: I carry a blanket

Step 2

If p and q or r, then s

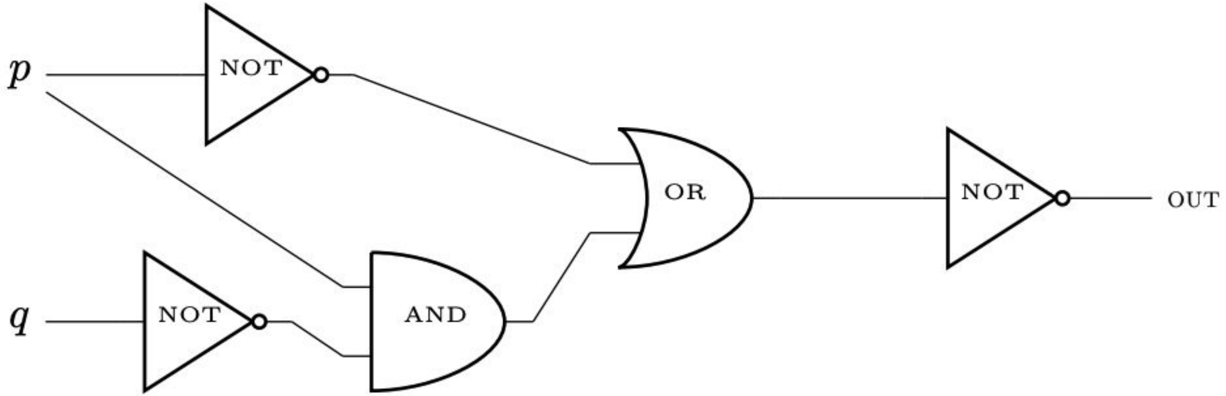
Step 3

$[(p \wedge q) \vee r] \rightarrow s$

Circuits

Problem 6 – Circuitous

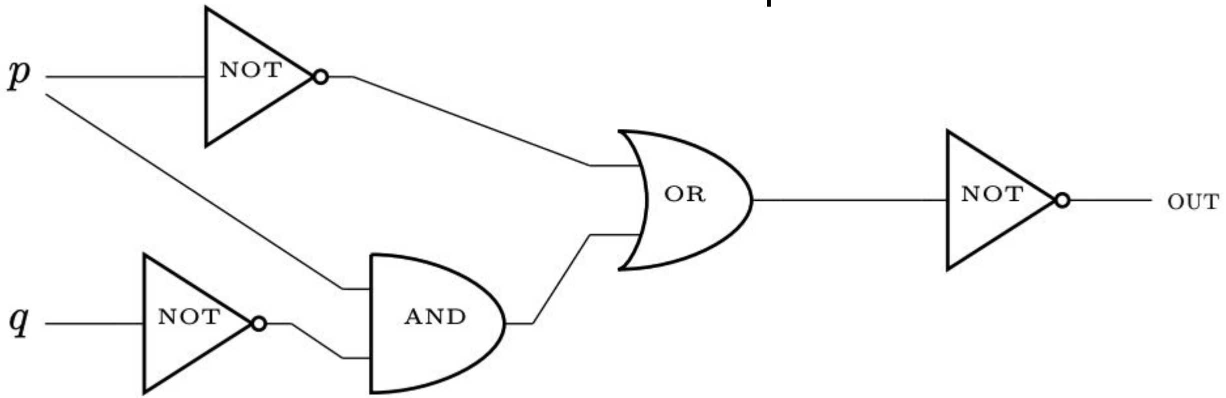
Translate the following circuit into a logical expression.



Work on this problem with the people around you, and then we'll go over it together!

Problem 6 – Circuitous

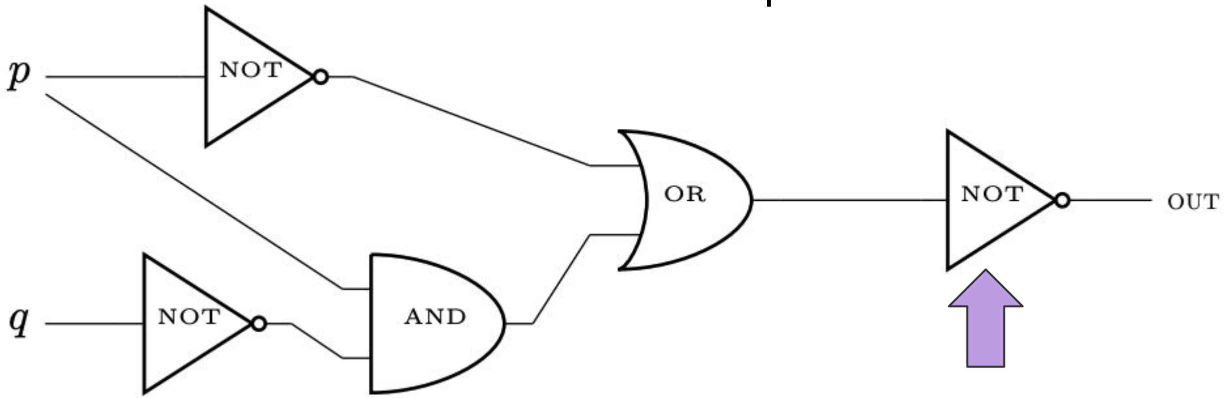
Translate the following circuit into a logical expression.



Tip: Think of starting from the end and working back to create the expression.

Problem 6 – Circuitous

Translate the following circuit into a logical expression.

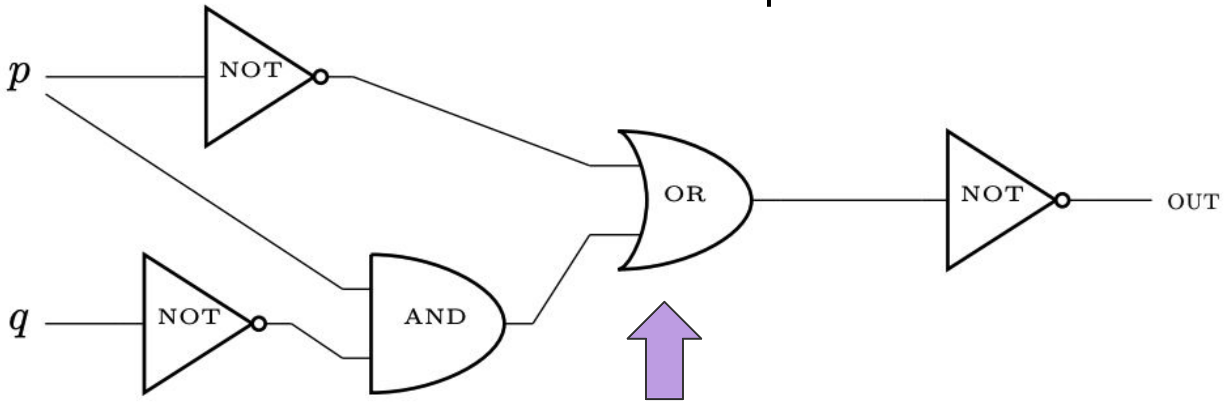


Tip: Think of starting from the end and working back to create the expression.

¬(...)

Problem 6 – Circuitous

Translate the following circuit into a logical expression.

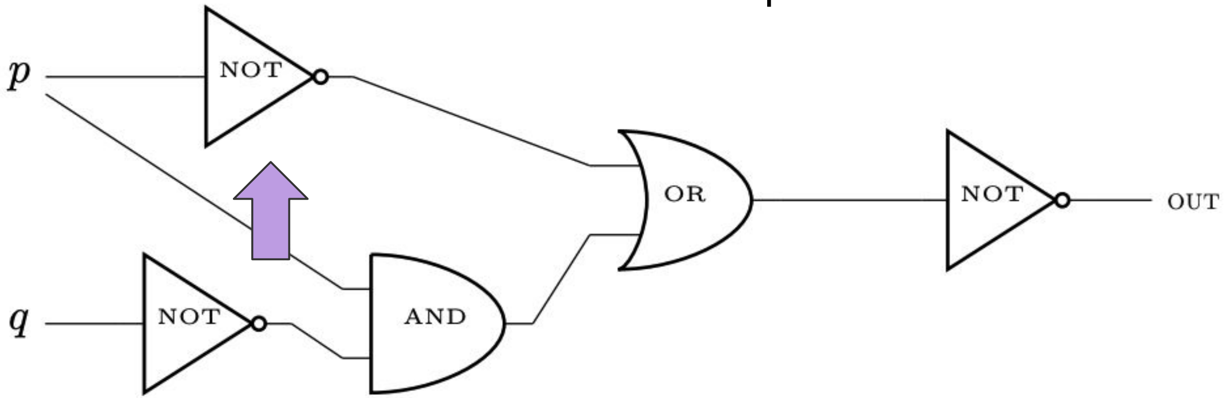


Tip: Think of starting from the end and working back to create the expression.

¬(... ∨ ...)

Problem 6 – Circuitous

Translate the following circuit into a logical expression.

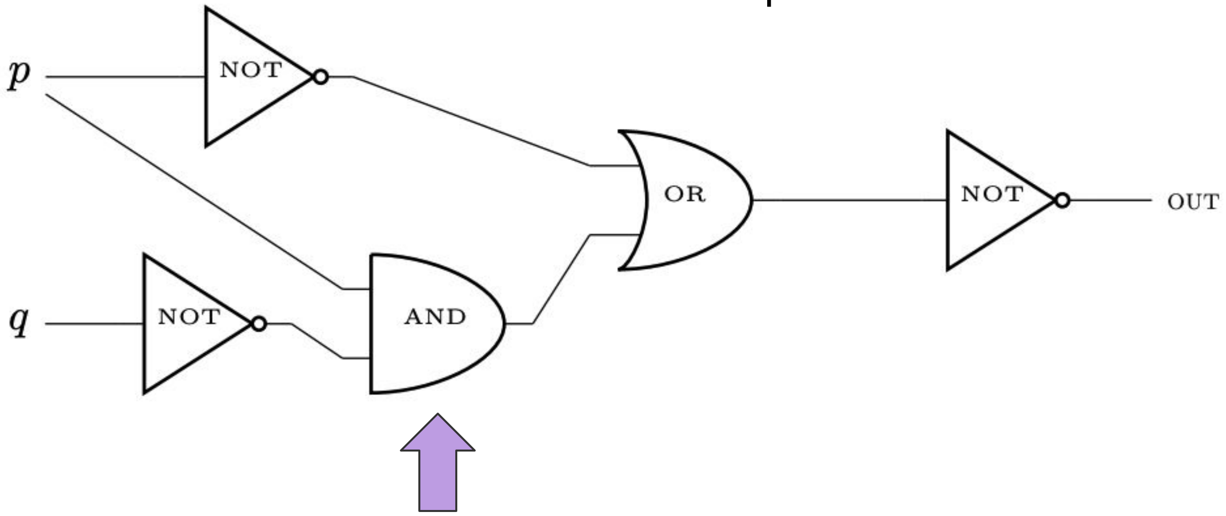


Tip: Think of starting from the end and working back to create the expression.

¬(¬ ... ∨ ...)

Problem 6 – Circuitous

Translate the following circuit into a logical expression.

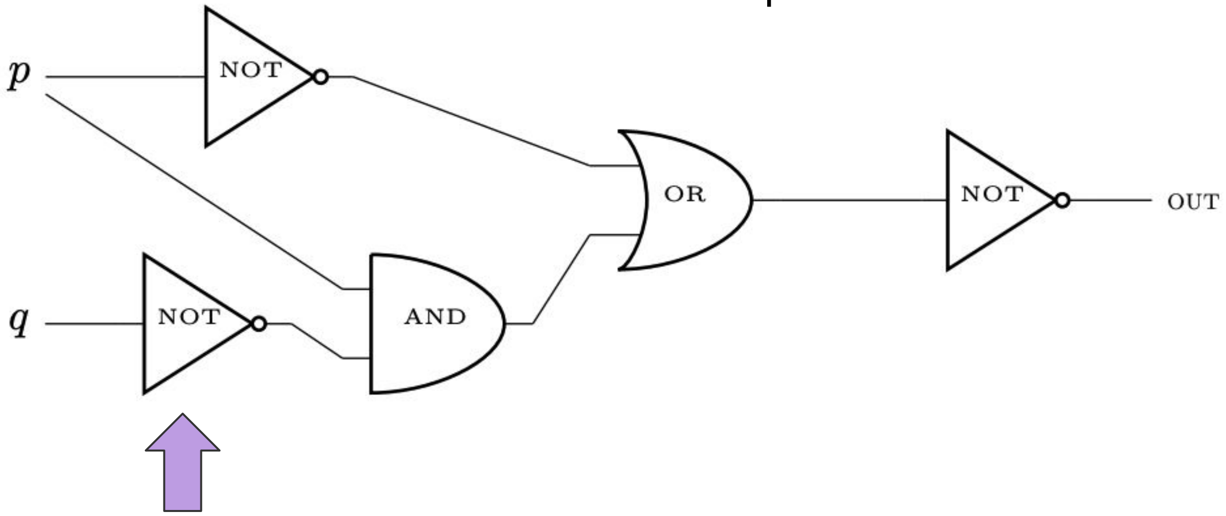


Tip: Think of starting from the end and working back to create the expression.

$\neg(\neg \ldots \vee (\ldots \wedge \ \ldots))$

Problem 6 – Circuitous

Translate the following circuit into a logical expression.

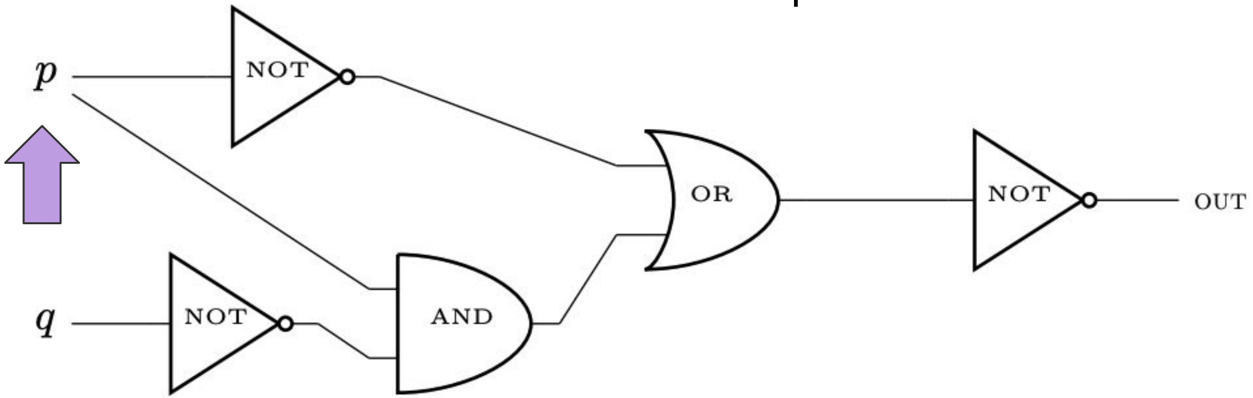


Tip: Think of starting from the end and working back to create the expression.

¬(¬ … ∨ (… ∧ ¬ …))

Problem 6 – Circuitous

Translate the following circuit into a logical expression.

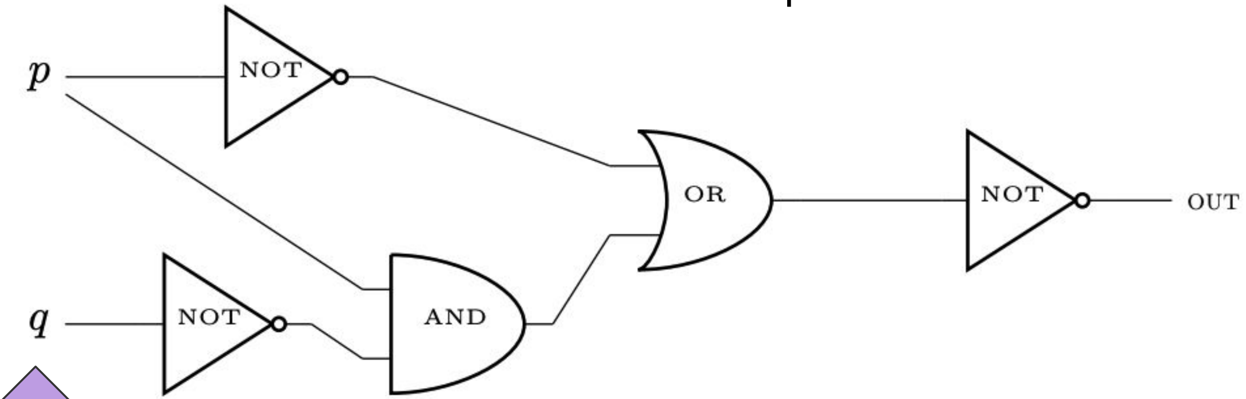


Tip: Think of starting from the end and working back to create the expression.

$\neg(\neg p \lor (p \land \neg \ldots))$

Problem 6 – Circuitous

Translate the following circuit into a logical expression.



Tip: Think of starting from the end and working back to create the expression.

$\neg(\neg p \vee (p \wedge \neg q))$

Normal Forms

(Canonical) Normal Forms

- Standard ways of translating a truth table into a proposition.
- We already did these in lecture when we translated implications into an expression only using ands, ors, and nots!
- Once you translate into one of these forms, **don’t simplify your expression any further!** It often looks like you can factor variables out to make it prettier, but the whole point is to write the expression into this standardized way, so just leave it as-is ☺

DNF (OR of ANDs)

- Disjunctive Normal Form
  - OR of ANDs
  - Method:
    1. Read the TRUE rows of the truth table
    2. AND together all the variable settings in a given (true) row
    3. OR together the true rows

DNF (OR of ANDs)

1. Read the TRUE rows of the truth table
2. AND together all the variable settings in a given (true) row
3. OR together the true rows

p	***q***	***G(p,q)***	
T	T	T	$p \wedge q$
T	F	F	
F	T	T	$\neg p \wedge q$
F	F	F	

$$G(p,q) \equiv (p \wedge q) \vee (\neg p \wedge q)$$

CNF (AND of ORs)

- Conjunctive Normal Form
  - AND of ORs
  - Method:
    1. Read the FALSE rows of the truth table
    2. OR together the negations of all the variable settings in the false row
    3. AND together the false rows

CNF (AND of ORs)

1. Read the FALSE rows of the truth table
2. OR together the negations of all the variable settings in the false row
3. AND together the false rows

p	*q*	*G(p,q)*	
T	T	T	
T	F	F	$\neg p \vee q$
F	T	T	
F	F	F	$p \vee q$

$$G(p, q) \equiv (\neg p \vee q) \wedge (p \vee q)$$

Problem 6(d)

Write the CNF and DNF expression for (b) **in Boolean Algebra**

r	q	$(r \vee q) \rightarrow (r \oplus q)$
T	T	F
T	F	T
F	T	T
F	F	T

Boolean Algebra: another notation for logic consisting of...

- a set of elements B = {0, 1}
- binary operations { + , • } (OR, AND)
- and a unary operation { ' } (NOT)

Problem 6(d)

Write the CNF and DNF expression for (b) **in Boolean Algebra**

r	q	(r v q) →(r ⊕ q)
T	T	F
T	F	T
F	T	T
F	F	T

Boolean Algebra: another notation for logic consisting of…

- a set of elements B = {0, 1}
- binary operations { + , • } (OR, AND)
- and a unary operation { ’ } (NOT)

Problem 6(d)

Write the CNF and DNF expression for (b) **in Boolean Algebra**

r	q	(r v q) →(r ⊕ q)
T	T	F
T	F	T
F	T	T
F	F	T

DNFs: (p Λ¬q) v (¬p Λ q) v (¬p Λ ¬q)

Problem 6(d)

Write the CNF and DNF expression for (b) **in Boolean Algebra**

r	q	(r v q) →(r ⊕ q)
T	T	F
T	F	T
F	T	T
F	F	T

DNFs: (p Λ¬q) v (¬p Λ q) v (¬p Λ ¬q)

Boolean Algebra: p · q’ + p’ · q + p’ · q’

Problem 6(d)

Write the CNF and DNF expression for (b) **in Boolean Algebra**

r	q	(r v q) →(r ⊕ q)
T	T	F
T	F	T
F	T	T
F	F	T

CNFs: (¬r v ¬q)

Problem 6(d)

Write the CNF and DNF expression for (b) **in Boolean Algebra**

r	q	(r v q) →(r ⊕ q)
T	T	F
T	F	T
F	T	T
F	F	T

CNFs: (¬r v ¬q)

Boolean Algebra: r’ + q’

That’s All, Folks!

Thanks for coming to section this week!
Any questions?