CSE 311 Section MR

Midterm Review

Administrivia

Announcements & Reminders

- HW6
  - Was due Wednesday 11/6
  - Late due date Saturday 11/9

- Midterm is Coming Next Week!!!
  - Wednesday 11/13 @ 6-7:30 pm in PAA A102 and A118
  - If you cannot make it, please let us know ASAP and we will schedule you for a makeup

Problem 1: Translation

Problem 1 – Translation

Let your domain of discourse be all coffee drinks. You should use the following predicates:

- soy(x) is true iff x contains soy milk.
- whole(x) is true iff x contains whole milk.
- sugar(x) is true iff x contains sugar
- decaf(x) is true iff x is not caffeinated.
- vegan(x) is true iff x is vegan.
- RobbieLikes(x) is true iff Robbie likes the drink x.

Translate each of the following statements into predicate logic. You may use quantifiers, the predicates above, and usual math connectors like $=$ and $\neq$.

a) Coffee drinks with whole milk are not vegan

b) Robbie only likes one coffee drink, and that drink is not vegan

c) There is a drink that has both sugar and soy milk.

Work on this problem with the people around you.

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(a) Coffee drinks with whole milk are not vegan.

(d) Translate the contrapositive of part (a) and write a matching (natural) English sentence.

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Translate the following symbolic logic statement into a (natural) English sentence. Take advantage of domain restriction.

$$\forall x([\text{decaf}(x) \land \text{RobbieLikes}(x)] \rightarrow \text{sugar}(x))$$

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$$\forall x([\text{decaf}(x) \wedge \text{RobbieLikes}(x)] \rightarrow \text{sugar}(x))$$

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$$\forall x([\texttt{decaf}(x) \land \texttt{RobbieLikes}(x)] \rightarrow \texttt{sugar}(x))$$

Write the negation of part (e) in predicate logic and translate it into a (natural) English sentence. Take advantage of domain restriction.

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Negate:

$$\forall x([\text{decaf}(x) \wedge \text{RobbieLikes}(x)] \rightarrow \text{sugar}(x))$$

Problem 2: Circuits and Normal Forms

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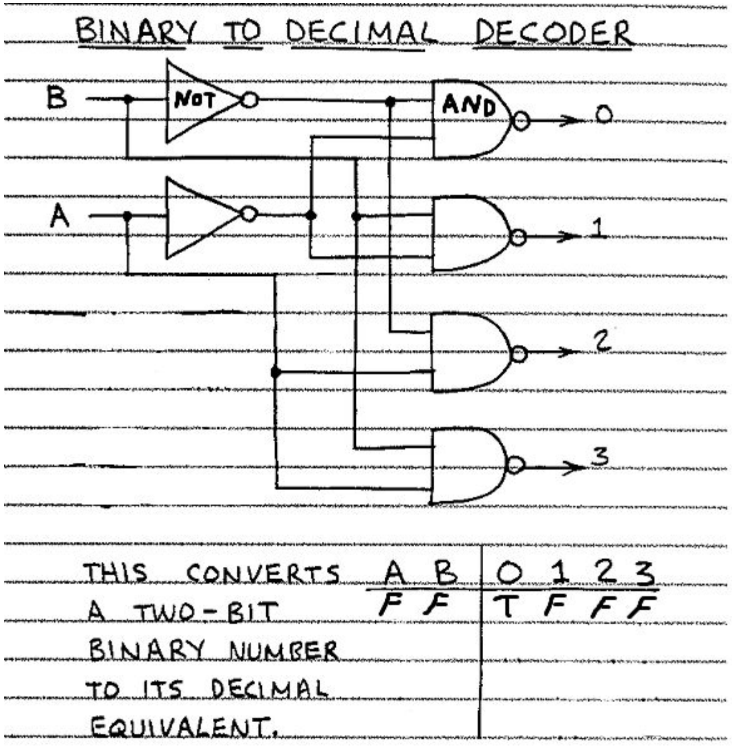


Here's a complex circuit.

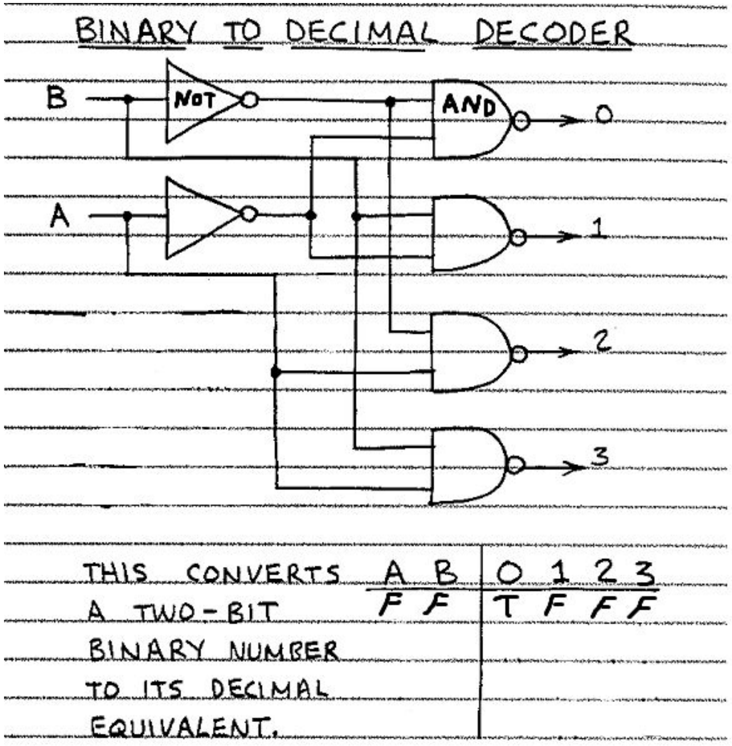
(a) DNF for the output "2"?

(b) CNF for the output "1"?

(c) DNF for the outputs greater than "1"?

(d) How does adding an input affect the outputs?

Problem 2 – Circuits and Normal Forms



Some tips:

- Complete the truth table
- Focus on one output at a time
- Recall CNF/DNF structure
- Eliminate answers

Problem 3: Boolean Algebra

Problem 3 – Boolean Algebra

Consider the following Boolean expression:

$$(A + A' \cdot B) \cdot (A + B)$$

(a) Simplify the given Boolean expression

(b) Identify whether the simplified expression is a tautology, contradiction, or neither

Work on this problem with the people around you.

Problem 4: Even Steven

Problem 4 – Even Steven

Prove that for all integers k, k(k +3) is even.
Recall that $\text{Even}(x) := \exists k(x = 2k)$ and $\text{Odd}(x) := \exists k(x = 2k + 1)$

(a) Let your domain be integers. Write the predicate logic of this claim.

(b) Write an English proof for this claim.

Problem 5: Number Theory

Problem 5 – Number Theory

Let p be a prime number at least 3 and let x be an integer such that $x^2 \% p = 1$.

a) Show that if an integer y satisfies $y \equiv 1 \pmod{p}$, then $y^2 \equiv 1 \pmod{p}$.

b) Repeat part (a), but don't use any theorems from the Number Theory Reference Sheet. That is, show the claim directly from the definitions.

c) From part (a), we can see that $x \% p$ can equal 1. Show that for any integer x, if $x^2 \equiv 1 \pmod{p}$, then $x \equiv 1 \pmod{p}$ or $x \equiv -1 \pmod{p}$. That is, show that the only value $x \% p$ can take other than 1 is $p - 1$.
Hint: Suppose you have an x such that $x^2 \equiv 1 \pmod{p}$ and use the fact that $x^2 - 1 = (x - 1)(x + 1)$
Hint: You may the following theorem without proof: if p is prime and $p \mid (ab)$ then $p \mid a$ or $p \mid b$.

Work on this problem with the people around you.

Problem 5 – Number Theory

Let p be a prime number at least 3 and let x be an integer such that $x^2 \% p = 1$

a) Show that if an integer y satisfies $y \equiv 1 \pmod{p}$, then $y^2 \equiv 1 \pmod{p}$.

Problem 5 – Number Theory

Let p be a prime number at least 3 and let x be an integer such that $x^2 \% p = 1$

b) Repeat part (a), but don't use any theorems from the Number Theory Reference Sheet. That is, show the claim directly from the definitions.

Problem 5 – Number Theory

Let p be a prime number at least 3 and let x be an integer such that $x^2 \% p = 1$

c) From part (a), we can see that $x\%p$ can equal 1. Show that for any integer x, if $x^2 \equiv 1 \pmod{p}$, then $x \equiv 1 \pmod{p}$ or $x \equiv -1 \pmod{p}$. That is, show that the only value $x\%p$ can take other than 1 is $p - 1$.
Hint: Suppose you have an x such that $x^2 \equiv 1 \pmod{p}$ and use the fact that
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Problem 6: Induction

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For any $n \in \mathbb{N}$, define S_n to be the sum of the squares of the first n positive integers, or $S_n = 1^2 + 2^2 + \cdots + n^2$.

Prove that for all $n \in \mathbb{N}$, $S_n = \frac{1}{6}n(n+1)(2n+1)$.

Work on this problem with the people around you.

Problem 7: Strong Induction

Problem 7 – Strong Induction

Robbie is planning to buy snacks for the members of his competitive roller-skating troupe. However, his local grocery store sells snacks in packs of 5 and packs of 7.

Prove that Robbie can buy exactly n snacks for all integers $n \geq 24$

Work on this problem with the people around you.

Problem 8: Wait, That Doesn't Add Up

Problem 8 - Wait, That Doesn't Add Up

Write a proof by contradiction for the following proposition: There exist no integers x and y such that $18x + 6y = 1$. In predicate logic this could be expressed as $\forall x \forall y (18x + 6y \neq 1)$. HINT: Try negating this statement before writing your proof.

Work on this problem with the people around you.

Problem 9: How Many Elements?

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9. How Many Elements?

For each of these, how many elements are in the set? If the set has infinitely many elements, say ∞.

(a) $A = \{1, 2, 3, 2\}$

(b) $B = \{\{\}, \{\{\}\}, \{\{\}, \{\}\}, \{\{\}, \{\}, \{\}\}, \dots\}$

(c) $C = A \times (B \cup \{7\})$

(d) $D = \varnothing$

(e) $E = \{\varnothing\}$

(f) $F = \mathcal{P}(\{\varnothing\})$

Work on this problem with the people around you.

Problem 10: Set = Set

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Prove the following set identities. Write both a formal inference proof **and** an English proof.

(a) Let the universal set be $\mathcal{U}$. Prove $A \cap \overline{B} \subseteq A \setminus B$ for any sets A, B.

(b) Prove that $(A \cap B) \times C \subseteq A \times (C \cup D)$ for any sets A, B, C, D.

Work on this problem with the people around you.

Problem 11: Set Equality

Problem 11 - Set Equality

Prove that $A \cap (A \cup B) = A$ for any sets A, B.

Work on this problem with the people around you.

That's All, Folks!

Thanks for coming to section this week!
Any questions?