CSE 311 Section 10

Final Review

Administrivia

Announcements & Reminders

- HW7 Regrade Requests
  - Grades out!
  - Submit a regrade request if something was graded incorrectly
- HW8
  - Due yesterday
  - Late due date 12/7
- Final Exam
  - Monday 12/9 @ 12:30pm-2:20 @ KNE 210/220
  - Fill out Form for Conflict Exam

Irregularity

A note for your final...

You **WILL** have a question on the final exam where you will have a choice between either **proving a language is irregular** OR **proving a set is uncountable**.

For section today, we will go over how to prove a language is irregular. There is also a problem in the handout on proving a set is uncountable you can review if you prefer to prepare for that question. You should pick whichever you think is easier for you, and make sure you are prepared to do it on the final exam!

Irregularity Template

Claim: L is an irregular language.

Proof: Suppose, for the sake of contradiction, that L is regular. Then there is a DFA M such that M accepts exactly L.

Let S = [TODO] (S is an infinite set of strings)
Because the DFA is finite, there are two (different) strings x, y in S such that x and y go to the same state when read by M. [TODO] (We don't get to choose x, y, but we can describe them based on that set S we just defined)

Consider the string z = [TODO] (We do get to choose z depending on x, y)

Since x, y led to the same state and M is deterministic, xz and yz will also lead to the same state q in M. Observe that xz = [TODO], so $xz \in L$ but yz = [TODO], so $yz \notin L$. Since q is can be only one of an accept or reject state, M does not actually recognize L. That's a contradiction!

Therefore, L is an irregular language.

Irregularity Example from Lecture

Claim: $\{0^k1^k : k \geq 0\}$ is an irregular language.

Proof: Suppose, for the sake of contradiction, that $L = \{0^k1^k : k \geq 0\}$ is regular. Then there is a DFA M such that M accepts exactly L.

Let $S = \{0^k : k \geq 0\}$
Because the DFA is finite, there are two (different) strings x, y in S such that x and y go to the same state when read by M. Since both are in S, $x = 0^a$ for some integer $a \geq 0$, and $y = 0^b$ for some integer $b \geq 0$, with $a \neq b$.

Consider the string $z = 1^a$.

Since x, y led to the same state and M is deterministic, xz and yz will also lead to the same state q in M. Observe that $xz = 0^a1^a$, so $xz \in L$ but $yz = 0^b1^a$, so $yz \notin L$. Since q is can be only one of an accept or reject state, M does not actually recognize L. That's a contradiction!

Therefore, L is an irregular language.

Problem 1 – Irregularity

a) Let $\Sigma = \{0, 1\}$. Prove that $\{0^n1^n0^n : n \geq 0\}$ is not regular.

b) Let $\Sigma = \{0, 1, 2\}$. Prove that $\{0^n(12)^m : n \geq m \geq 0\}$ is not regular.

Work on this problem with the people around you.

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Let S = [TODO]
Because the DFA is finite, there are two (different) strings x, y in S such that x and y go to the same state when read by M. [TODO] .

Consider the string z = [TODO] .

Since x, y led to the same state and M is deterministic, xz and yz will also lead to the same state q in M. Observe that xz = [TODO] , so $xz \in L$ but yz = [TODO] , so $yz \notin L$. Since q is can be only one of an accept or reject state, M does not actually recognize L. That's a contradiction!

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Let $S = \{0^n1^n : n \geq 0\}$
Because the DFA is finite, there are two (different) strings x, y in S such that x and y go to the same state when read by M. [TODO] .

Consider the string z = [TODO] .

Since x, y led to the same state and M is deterministic, xz and yz will also lead to the same state q in M. Observe that xz = [TODO] , so $xz \in L$ but yz = [TODO] , so $yz \notin L$. Since q is can be only one of an accept or reject state, M does not actually recognize L. That's a contradiction!

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Because the DFA is finite, there are two (different) strings x, y in S such that x and y go to the same state when read by M. Since both are in S, $x = 0^a1^a$ for some integer $a \geq 0$, and $y = 0^b1^b$ for some integer $b \geq 0$, with $a \neq b$.

Consider the string z = [TODO] .

Since x, y led to the same state and M is deterministic, xz and yz will also lead to the same state q in M. Observe that xz = [TODO] , so $xz \in L$ but yz = [TODO] , so $yz \notin L$. Since q is can be only one of an accept or reject state, M does not actually recognize L. That's a contradiction!

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Consider the string $z = 0^a$.

Since x, y led to the same state and M is deterministic, xz and yz will also lead to the same state q in M. Observe that xz = [TODO] , so $xz \in L$ but yz = [TODO] , so $yz \notin L$. Since q is can be only one of an accept or reject state, M does not actually recognize L. That's a contradiction!

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(b) Let $\Sigma = \{0, 1, 2\}$. Prove that $\{0^n(12)^m : n \geq m \geq 0\}$ is not regular.

Claim: $\{0^n(12)^m : n \geq m \geq 0\}$ is an irregular language.

Proof: Suppose, for the sake of contradiction, that $L = \{0^n(12)^m : n \geq m \geq 0\}$ is regular. Then there is a DFA M such that M accepts exactly L.

Let S = [TODO]
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Consider the string $z = (12)^a$.

Since x, y led to the same state and M is deterministic, xz and yz will also lead to the same state q in M. Observe that xz = [TODO] , so $xz \in L$ but yz = [TODO] , so $yz \notin L$. Since q is can be only one of an accept or reject state, M does not actually recognize L. That's a contradiction!

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Since x, y led to the same state and M is deterministic, xz and yz will also lead to the same state q in M. Observe that $xz = 0^a(12)^a$, so $xz \in L$ but $yz = 0^b(12)^a$, so $yz \notin L$. Since q is can be only one of an accept or reject state, M does not actually recognize L. That's a contradiction!

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Problem 11– Review: Uncountability

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Suppose for the sake of contradiction that S is countable. Then there exists a surjection $f : \mathbb{N} \to S$. So for each natural number i, we have some decimal sequence of 0s and 1s that i maps to.

We now construct an element x. We start x with 0. (a zero and decimal point). Then for all $i \in \mathbb{N}$, let the ith digit after the decimal point be 1 if $f(i) = 0$, and 0 if $f(i) = 1$.

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Note that by our construction, for any $i \in N$, $f(i)$ differs from x on the i-th digit after the decimal point. Furthermore by our construction, x contains only 0s and 1s in its decimal expansion and $x \in [0, 1)$, so $x \in S$. Since $x \in S$ but is not in the range of f, f is not surjective. This is a contradiction. Therefore S is uncountable.

Final Review

Problem 5 – Review: Translations

Translate the following sentences into logical notation if the English statement is given or to an English statement if the logical statement is given, taking into account the domain restriction. Let the domain of discourse be students and courses. Use predicates `Student`, `Course`, `CseCourse` to do the domain restriction. You can use `Taking(x, y)` which is true if and only if x is taking y. You can also use `RobbieTeaches(x)` if and only if Robbie teaches x and `ContainsTheory(x)` if and only if x contains theory. Find the contrapositive and contradiction for questions (a) - (c).

(a) Every student is taking some course.

(b) There is a student that is not taking every cse course.

(c) Some student is taking only one cse course.

(d) ∀x[(Course(x) ∧ RobbieTeaches(x)) → ContainsTheory(x)]

(e) ∃x CseCourse(x) ∧ RobbieTeaches(x) ∧ ContainsTheory(x) ∧
∀y((CseCourse(y) ∧ RobbieTeaches(y)) → x = y)

Work on this problem with the people around you.

Problem 5 – Review: Translations

a) **Every student is taking some course.**

b) There is a student that is not taking any cse course.

c) Some student is taking only one cse course.

d) $\forall x[(Course(x) \land RobbieTeaches(x)) \rightarrow ContainsTheory(x)]$

e) $\exists x\ CseCourse(x) \land RobbieTeaches(x) \land ContainsTheory(x) \land \forall y((CseCourse(y) \land RobbieTeaches(y)) \rightarrow x = y)$

Problem 5 – Review: Translations

a) Every student is taking some course.

Translation:

$\forall x \exists y(\text{Student}(x) \rightarrow [\text{Course}(y) \wedge \text{Taking}(x, y)])$

Problem 5 – Review: Translations

a) Every student is taking some course.

Translation: $\forall x \exists y(\text{Student}(x) \rightarrow [\text{Course}(y) \land \text{Taking}(x, y)])$

Contrapositive: $\forall x \exists y((\neg \text{Course}(y) \lor \neg \text{Taking}(x, y)) \rightarrow \neg \text{Student}(x))$

Consider the implication

$\text{Student}(x) \rightarrow [\text{Course}(y) \land \text{Taking}(x, y)]$

Take the contrapositive: $P \rightarrow Q$ is $\neg Q \rightarrow \neg P$

$\neg(\text{Course}(y) \land \text{Taking}(x, y)) \rightarrow \neg \text{Student}(x)$

Apply DeMorgan's

$(\neg \text{Course}(y) \lor \neg \text{Taking}(x, y)) \rightarrow \neg \text{Student}(x)$

Apply Quantifiers

$\forall x \exists y((\neg \text{Course}(y) \lor \neg \text{Taking}(x, y)) \rightarrow \neg \text{Student}(x))$

Problem 5 – Review: Translations

a) Every student is taking some course.

Translation: $\forall x \exists y(\text{Student}(x) \rightarrow [\text{Course}(y) \land \text{Taking}(x, y)])$

Contrapositive: $\forall x \exists y((\neg \text{Course}(y) \lor \neg \text{Taking}(x, y)) \rightarrow \neg \text{Student}(x))$

Contradiction: $\exists x \forall y(\neg \text{Student}(x) \land [\neg \text{Course}(y) \lor \neg \text{Taking}(x, y)])$

Negate the statement

$\neg \forall x \exists y(\text{Student}(x) \rightarrow [\text{Course}(y) \land \text{Taking}(x, y)])$

Apply negation to quantifiers

$\exists x \forall y(\neg (\text{Student}(x) \rightarrow [\text{Course}(y) \land \text{Taking}(x, y)]))$

Apply the law of implication: $A \rightarrow B \equiv \neg A \lor B$

$\exists x \forall y(\neg (\neg \text{Student}(x) \lor [\text{Course}(y) \land \text{Taking}(x, y)]))$

Negate the statement & apply DeMorgan's law

$\exists x \forall y(\neg \text{Student}(x) \land [\neg \text{Course}(y) \lor \neg \text{Taking}(x, y)])$

Problem 5 – Review: Translations

a) Every student is taking some course.

Translation: $\forall x \exists y(\text{Student}(x) \rightarrow [\text{Course}(y) \land \text{Taking}(x, y)])$
Contrapositive: $\forall x \exists y((\neg \text{Course}(y) \lor \neg \text{Taking}(x, y)) \rightarrow \neg \text{Student}(x))$
Contradiction: $\exists x \forall y(\neg \text{Student}(x) \land [\neg \text{Course}(y) \lor \neg \text{Taking}(x, y)])$

Problem 5 – Review: Translations

a) Every student is taking some course.

b) There is a student that is not taking any cse course.

c) Some student is taking only one cse course.

d) $\forall x[(\text{Course}(x) \land \text{RobbieTeaches}(x)) \rightarrow \text{ContainsTheory}(x)]$

e) $\exists x\ \text{CseCourse}(x) \land \text{RobbieTeaches}(x) \land \text{ContainsTheory}(x) \land \forall y((\text{CseCourse}(y) \land \text{RobbieTeaches}(y)) \rightarrow x = y)$

Problem 5 – Review: Translations

b) There is a student that is not taking any cse course.

Translation:

$\exists x \forall y[\text{Student}(x) \land (\text{CseCourse}(y) \rightarrow \neg \text{Taking}(x, y))]$

Problem 5 – Review: Translations

b) There is a student that is not taking any cse course.

Translation: $\exists x \forall y[Student(x) \land (CseCourse(y) \rightarrow \neg Taking(x, y))]$

Contrapositive: No contrapositive

You can only meaningfully apply contrapositives to implications.

There is no implication on our logical predicate.

Problem 5 – Review: Translations

b) There is a student that is not taking any cse course.

Translation: $\exists x \forall y[Student(x) \land (CseCourse(y) \rightarrow \neg Taking(x, y))]$
Contrapositive: No contrapositive

Contradiction $\forall x \exists y(\neg Student(x) \lor [CseCourse(y) \land Taking(x, y)])$

Negate the statement

$\neg \exists x \forall y[Student(x) \land (CseCourse(y) \rightarrow \neg Taking(x, y))]$

Apply negation to quantifiers

$\forall x \exists y \neg[Student(x) \land (CseCourse(y) \rightarrow \neg Taking(x, y))]$

Apply the law of implication: $A \rightarrow B \equiv \neg A \lor B$

$\forall x \exists y \neg[Student(x) \land (\neg CseCourse(y) \lor \neg Taking(x, y))]$

Negate the statement & apply DeMorgan's law

$\forall x \exists y \neg Student(x) \lor (CseCourse(y) \land Taking(x, y))$

Problem 5 – Review: Translations

b) There is a student that is not taking any cse course.

Translation: $\exists x \forall y[Student(x) \land (CseCourse(y) \rightarrow \neg Taking(x, y))]$
Contrapositive: No contrapositive
Contradiction: $\forall x \exists y(\neg Student(x) \lor [CseCourse(y) \land Taking(x, y)])$

Problem 5 – Review: Translations

a) Every student is taking some course.

b) There is a student that is not taking any cse course.

c) **Some student is taking only one cse course.**

d) $\forall x[(Course(x) \land RobbieTeaches(x)) \rightarrow ContainsTheory(x)]$

e) $\exists x\ CseCourse(x) \land RobbieTeaches(x) \land ContainsTheory(x) \land \forall y((CseCourse(y) \land RobbieTeaches(y)) \rightarrow x = y)$

Problem 5 – Review: Translations

c) Some student is taking only one cse course.

Translation:

$\exists x \exists y[Student(x) \land CseCourse(y) \land Taking(x, y) \land \forall z((CseCourse(z) \land Taking(x, z)) \rightarrow y = z))]$

Problem 5 – Review: Translations

c) Some student is taking only one cse course.

Translation: $\exists x \exists y[\text{Student}(x) \land \text{CseCourse}(y) \land \text{Taking}(x, y) \land \forall z((\text{CseCourse}(z) \land \text{Taking}(x, z)) \rightarrow y$

Contrapositive: No contrapositive

You can only meaningfully apply contrapositives to implications.

There is no implication on our logical predicate.

Problem 5 – Review: Translations

c) Some student is taking only one cse course.

Translation: $\exists x \exists y[Student(x) \land CseCourse(y) \land Taking(x, y) \land \forall z((CseCourse(z) \land Taking(x, z)) \rightarrow y$

Contrapositive: No contrapositive

Contradiction: $\forall x \forall y\, [\neg Student(x) \lor \neg CseCourse(y) \lor \neg Taking(x, y) \lor \exists z(CseCourse(z) \land Taking(x, z$
$(y \neq z))]$

Negate the statement

$\neg \exists x \exists y[Student(x) \land CseCourse(y) \land Taking(x, y) \land \forall z((CseCourse(z) \land Taking(x, z)) \rightarrow y = z))]$

Apply negation to quantifiers

$\forall x \forall y\, \neg[Student(x) \land CseCourse(y) \land Taking(x, y) \land \forall z((CseCourse(z) \land Taking(x, z)) \rightarrow y = z))]$

Apply negation & DeMorgan's law

$\forall x \forall y\, [\neg Student(x) \lor \neg CseCourse(y) \lor \neg Taking(x, y) \lor \neg(\forall z((CseCourse(z) \land Taking(x, z)) \rightarrow y = z)))]$

Problem 5 – Review: Translations

c) Some student is taking only one cse course.

Translation: $\exists x \exists y[Student(x) \land CseCourse(y) \land Taking(x, y) \land \forall z((CseCourse(z) \land Taking(x, z)) \rightarrow y$

Contrapositive: No contrapositive

Contradiction: $\forall x \forall y\ [\neg Student(x) \lor \neg CseCourse(y) \lor \neg Taking(x, y) \lor \exists z(CseCourse(z) \land Taking(x, z$
$(y \neq z))]$

$\forall x \forall y\ [\neg Student(x) \lor \neg CseCourse(y) \lor \neg Taking(x, y) \lor \neg(\forall z((CseCourse(z) \land Taking(x, z)) \rightarrow y = z)))]$

Negate the quantifier for z

$\forall x \forall y\ [\neg Student(x) \lor \neg CseCourse(y) \lor \neg Taking(x, y) \lor \exists z(\neg((CseCourse(z) \land Taking(x, z)) \rightarrow y = z))]$

Apply law of implication: $A \rightarrow B \equiv \neg A \lor B$

$\forall x \forall y\ [\neg Student(x) \lor \neg CseCourse(y) \lor \neg Taking(x, y) \lor \exists z(\neg(\neg(CseCourse(z) \land Taking(x, z)) \lor (y = z))]$

Problem 5 – Review: Translations

c) Some student is taking only one cse course.

Translation: $\exists x \exists y[Student(x) \land CseCourse(y) \land Taking(x, y) \land \forall z((CseCourse(z) \land Taking(x, z)) \rightarrow y$

Contrapositive: No contrapositive

Contradiction: $\forall x \forall y\, [\neg Student(x) \lor \neg CseCourse(y) \lor \neg Taking(x, y) \lor \exists z(CseCourse(z) \land Taking(x, z$
$(y \neq z))]$

$\forall x \forall y\, [\neg Student(x) \lor \neg CseCourse(y) \lor \neg Taking(x, y) \lor \exists z(\neg(\neg(CseCourse(z) \land Taking(x, z))$
$\lor$
$(y = z))]$

Apply negation & DeMorgan's law

$\forall x \forall y\, [\neg Student(x) \lor \neg CseCourse(y) \lor \neg Taking(x, y) \lor \exists z(CseCourse(z) \land Taking(x, z)) \land$
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Problem 5 – Review: Translations

c) Some student is taking only one cse course.

Translation: $\exists x \exists y[Student(x) \land CseCourse(y) \land Taking(x, y) \land \forall z((CseCourse(z) \land Taking(x, z)) \rightarrow y$

Contrapositive: No contrapositive

Contradiction: $\forall x \forall y\, [\neg Student(x) \lor \neg CseCourse(y) \lor \neg Taking(x, y) \lor \exists z(CseCourse(z) \land Taking(x, z$
$(y \neq z))]$

Problem 5 – Review: Translations

a) Every student is taking some course.

b) There is a student that is not taking any cse course.

c) Some student is taking only one cse course.

d) $\forall x[(Course(x) \land RobbieTeaches(x)) \rightarrow ContainsTheory(x)]$

e) $\exists x\ CseCourse(x) \land RobbieTeaches(x) \land ContainsTheory(x) \land \forall y((CseCourse(y) \land RobbieTeaches(y)) \rightarrow x = y)$

Problem 5 – Review: Translations

d) $\forall x[(Course(x) \land RobbieTeaches(x)) \rightarrow ContainsTheory(x)]$

Every course taught by Robbie contains theory.

e) $\exists x\, CseCourse(x) \land RobbieTeaches(x) \land ContainsTheory(x) \land \forall y((CseCourse(y) \land RobbieTeaches(y)) \rightarrow x = y)$

Problem 5 – Review: Translations

d) $\forall x[(Course(x) \land RobbieTeaches(x)) \rightarrow ContainsTheory(x)]$

Every course taught by Robbie contains theory.

e) $\exists x\, CseCourse(x) \land RobbieTeaches(x) \land ContainsTheory(x) \land \forall y((CseCourse(y) \land RobbieTeaches(y)) \rightarrow x = y)$

There is only one cse course that Robbie teaches and that course contains theory.

Problem 6 – Review: Proof Skeleton Setup

For each of the following, write the beginning and target of your proof (not the middle reasoning)

a) Contradiction — For all real numbers x, y, if $x \neq y, x > 0, y > 0$, then $\frac{x}{y} + \frac{y}{x} > 2$.

b) Contrapositive — every multiple of 3 can be written as a sum of three consecutive integers.

c) Direct Proof — $n^2 - 3$ is even if n is odd, for some integer n.

Work on this problem with the people around you.

Problem 6 – Review: Proof Skeleton Setup

For each of the following, write the beginning and target of your proof (not the middle reasoning)

Contradiction   For all real numbers x, y, if $x \neq y, x > 0, y > 0$, then $\frac{x}{y} + \frac{y}{x} > 2$.

(i) How would you start your proof by contradiction? Remember to introduce all variables needed and all starting assumptions.

Work on this problem with the people around you.

Problem 6 – Review: Proof Skeleton Setup

For each of the following, write the beginning and target of your proof (not the middle reasoning)

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(i) How would you start your proof by contradiction? Remember to introduce all variables needed and all starting assumptions.

Solution:

Recall that contradiction assumes the negation of the statement is true, then shows that this leads to a contradiction.

Our premise is a universal for all statement, thus the negation is an existential there exists statement. As we bring the negation in, we do not adjust the definitions of x, y, but we do negate the conclusion: $>$ becomes $\leq$.

Suppose, for the sake of contradiction, there exist some real numbers x, y, $x \neq y, x > 0, y > 0$ and $\frac{x}{y} + \frac{y}{x} \leq 2$.

Work on this problem with the people around you.

Problem 6 – Review: Proof Skeleton Setup

For each of the following, write the beginning and target of your proof (not the middle reasoning)

Contradiction For all real numbers x, y, if $x \neq y, x > 0, y > 0$, then $\frac{x}{y} + \frac{y}{x} > 2$.

(ii) What would your target be? Do not write the full proof. If the target is unclear, describe the statement you should target instead.

Work on this problem with the people around you.

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Contradiction   For all real numbers x, y, if $x \neq y, x > 0, y > 0$, then $\frac{x}{y} + \frac{y}{x} > 2$.

(ii) What would your target be? Do not write the full proof. If the target is unclear, describe the statement you should target instead.

Solution:

But, this is a contradiction, because ...

The target is to show there is a contradiction based on the negated claim.

Work on this problem with the people around you.

Problem 6 – Review: Proof Skeleton Setup

For each of the following, write the beginning and target of your proof (not the middle reasoning)

Contrapositive — every multiple of 3 can be written as a sum of three consecutive integers.

(i) How would you start your proof by contrapositive? Remember to introduce all variables needed and all starting assumptions.

Work on this problem with the people around you.

Problem 6 – Review: Proof Skeleton Setup

For each of the following, write the beginning and target of your proof (not the middle reasoning)

Contrapositive every multiple of 3 can be written as a sum of three consecutive integers.

(i) How would you start your proof by contrapositive? Remember to introduce all variables needed and all starting assumptions.

Solution:

The original statement resembled, for some integer a, if a is a multiple of 3 then a can be expressed as the sum of three consecutive integers $(b) + (b+1) + (b+2)$.

The contrapositive negates the statement, which reverses the implication and negates both sides: If a can NOT be expressed as the sum of three consecutive integers $(b) + (b+1) + (b+2)$, then a is NOT a multiple of three.

Work on this problem with the people around you.

Problem 6 – Review: Proof Skeleton Setup

For each of the following, write the beginning and target of your proof (not the middle reasoning)

Contrapositive   every multiple of 3 can be written as a sum of three consecutive integers.

(ii) What would your target be? Do not write the full proof. If the target is unclear, describe the statement you should target instead.

Work on this problem with the people around you.

Problem 6 – Review: Proof Skeleton Setup

For each of the following, write the beginning and target of your proof (not the middle reasoning)

Contrapositive every multiple of 3 can be written as a sum of three consecutive integers.

(ii) What would your target be? Do not write the full proof. If the target is unclear, describe the statement you should target instead.

Solution:

Since our three consecutive integers $b, b+1, b+2$ were arbitrary, we know a can NOT be expressed as the sum of three consecutive integers $(b)+(b+1)+(b+2)$, and therefore a is NOT a multiple of three.

Work on this problem with the people around you.

Problem 6 – Review: Proof Skeleton Setup

For each of the following, write the beginning and target of your proof (not the middle reasoning)

Direct Proof $n^2 - 3$ is even if n is odd, for some integer n.

(i) How would you start your direct proof? Remember to introduce all variables needed and all starting assumptions.

Work on this problem with the people around you.

Problem 6 – Review: Proof Skeleton Setup

For each of the following, write the beginning and target of your proof (not the middle reasoning)

Direct Proof $\quad$ $n^2 - 3$ is even if n is odd, for some integer n.

(i) How would you start your direct proof? Remember to introduce all variables needed and all starting assumptions.

Solution:

The hypothesis, the premise of our direct proof implication, is that n is odd.

Work on this problem with the people around you.

Problem 6 – Review: Proof Skeleton Setup

For each of the following, write the beginning and target of your proof (not the middle reasoning)

Direct Proof $n^2 - 3$ is even if n is odd, for some integer n.

(ii) What would your target be? Do not write the full proof. If the target is unclear, describe the statement you should target instead.

Work on this problem with the people around you.

Problem 6 – Review: Proof Skeleton Setup

For each of the following, write the beginning and target of your proof (not the middle reasoning)

Direct Proof $n^2 - 3$ is even if n is odd, for some integer n.

(ii) What would your target be? Do not write the full proof. If the target is unclear, describe the statement you should target instead.

Solution:

The conclusion of our direct proof is that therefore, $n^2 - 3$ is even when n is odd.

Work on this problem with the people around you.

Problem 7 – Review: Set Theory

Suppose that $A \subseteq B$. Prove that $\mathcal{P}(A) \subseteq \mathcal{P}(B)$.

Work on this problem with the people around you.

Problem 7 – Review: Set Theory

Suppose that $A \subseteq B$. Prove that $\mathcal{P}(A) \subseteq \mathcal{P}(B)$.

Suppose $A \subseteq B$. Let $X \in \mathcal{P}(A)$ be an arbitrary element. Then by definition of powerset, $X \subseteq A$. Let $y \in X$ be arbitrary. Then since $X \subseteq A$, by definition of subset, $y \in A$. Since $A \subseteq B$, by definition of subset again, $y \in B$. Since y was arbitrary in X, by definition of subset once more, $X \subseteq B$. Then by definition of powerset, $X \in \mathcal{P}(B)$. Since X was arbitrary in $P(A)$, we have shown $\mathcal{P}(A) \subseteq P(B)$.

Work on this problem with the people around you.

Problem 8 – Review: Functions

Let $f : X \to Y$ be a function. For a subset C of X, define f(C) to be the set of elements that f sends C to. In other words, $f(C) = \{f(c) : c \in C\}$.

Let A, B be subsets of X. Prove that $f(A \cap B) \subseteq f(A) \cap f(B)$.

Work on this problem with the people around you.

Problem 8 – Review: Functions

Let $f : X \rightarrow Y$ be a function. For a subset C of X, define $f(C)$ to be the set of elements that f sends C to. In other words, $f(C) = \{f(c) : c \in C\}$.

Let A, B be subsets of X. Prove that $f(A \cap B) \subseteq f(A) \cap f(B)$.

Let $y \in f(A \cap B)$ be arbitrary.

Then there exists some element $x \in A \cap B$ such that $f(x) = y$.
Then by the definition of intersection, $x \in A$ and $x \in B$. Then $f(x) \in f(A)$ and $f(x) \in f(B)$. Thus $y \in f(A)$ and $y \in f(B)$.

By definition of intersection, $y \in f(A) \cap f(B)$.

Since y was arbitrary, $f(A \cap B) \subseteq f(A) \cap f(B)$.

Problem 9 – Review: Induction

a) A Husky Tree is a tree built by the following definition:

Basis: A single gold node is a Husky Tree.
Recursive Rules:
1. Let T1, T2 be two Husky Trees, both with root nodes colored gold. Make a new purple root node and attach the roots of T1, T2 to the new node to make a new Husky Tree.
2. Let T1, T2 be two Husky Trees, both with root nodes colored purple. Make a new purple root node and attach the roots of T1, T2 to the new node to make a new Husky Tree.
3. Let T1, T2 be two Husky Trees, one with a purple root, the other with a gold root. Make a new gold root node, and attach the roots of T1, T2 to the new node to make a new Husky Tree.

Use structural induction to show that for every Husky Tree: if it has a purple root, then it has an even number of leaves and if it has a gold root, then it has an odd number of leaves.

Work on this problem with the people around you.

Problem 9 – Review: Induction

1. Define $P()$ Show that $P(x)$ holds for all $x \in S$. State your proof is by structural induction.

2. Base Case: Show $P(x)$ for all base cases x in S.

3. Inductive Hypothesis: Suppose $P(x)$ for all x listed as in S in the recursive rules.

4. Inductive Step: Show $P()$ holds for the "new element" given.

You will need a separate step for every rule.

5. Therefore $P(x)$ holds for all $x \in S$ by the principle of induction.

Problem 9 – Review: Induction

Let P(T) be "if T has a purple root, then it has an even number of leaves and if T has a gold root, then it has an odd number of leaves." We show P(T) holds for all Husky Trees T by structural induction.

Base Case: Show P(•). Let • be a Husky Tree made from the basis step. By the definition of Husky Tree, • must be a single gold node. That node is also a leaf node (since it has no children), so there are an odd number (specifically, 1) of leaves, as required for a gold root node. So, P(•) holds.

Inductive Hypothesis: Suppose P(T1) and P(T2) for arbitrary Husky Trees T1 and T2 .

Problem 9 – Review: Induction

Inductive Step: Show $P(Y)$ holds: We will have separate cases for each possible rule.
Rule 1: Suppose T1 and T2 both have gold roots. By the recursive rule, Y has a purple root. By inductive hypothesis on T1, since T1's root is gold, it has an odd number of leaves. Similarly by IH, T2 has an odd number of leaves. Y's leaves are exactly the leaves of T1 and T2, so the total number of leaves in Y is the sum of two odd numbers, which is even. Thus Y has an even number of leaves, as is required for a purple root. Thus P(Y) holds.
Rule 2: Suppose T1 and T2 both have purple roots. By the recursive rule, Y has a purple root. By inductive hypothesis on T1, since T1's root is purple, it has an even number of leaves. Similarly by IH, T2 has an even number of leaves. Y's leaves are exactly the leaves of T1 and T2, so the total number of leaves in Y is the sum of two even numbers, which is even. Thus Y has an even number of leaves, as is required for a purple root. Thus P(Y) holds.
Rule 3: Suppose T1 and T2 have opposite colored roots. Let T1 be the one with a gold root, and T2 the one with the purple root. By the recursive rule, Y has a gold root. By inductive hypothesis on T1, since T1's root is gold, it has an odd number of leaves. Similarly, by IH, T2 has an even number of leaves since it has a purple root. Y's leaves are exactly the leaves of T1 and T2, so the total number of leaves in Y is the sum of an odd number and an even number, which is odd. Thus Y has an odd number of leaves, as is required for a gold root. Thus P(T) holds.

Therefore $P(T)$ holds for all Husky Trees *T* by the principle of induction.

Problem 9 – Review: Induction

(b) Use induction to prove that for every positive integer n, $1 + 5 + 9 + \cdots + (4n - 3) = n(2n - 1)$

Work on this problem with the people around you.

Problem 9 – Review: Induction

(b) Use induction to prove that for every positive integer n, $1 + 5 + 9 + \cdots + (4n - 3) = n(2n - 1)$

Let $P(n)$ be "". We show $P(n)$ holds for (some) n by induction on n.

Base Case: $P(b)$:

Inductive Hypothesis: Suppose $P(k)$ holds for an arbitrary $k \geq b$.

Inductive Step: Goal: Show $P(k+1)$:

Conclusion: Therefore, $P(n)$ holds for (some) n by the principle of induction.

Problem 9 – Review: Induction

(b) Use induction to prove that for every positive integer n, $1 + 5 + 9 + \cdots + (4n - 3) = n(2n - 1)$

Let $P(n)$ be "$1 + 5 + 9 + \cdots + (4n - 3) = n(2n - 1)$". We show $P(n)$ holds for all $n \in \mathbb{Z}^+$ by induction on n.

Base Case: $P(b)$:

Inductive Hypothesis: Suppose $P(k)$ holds for an arbitrary $k \geq b$.

Inductive Step: Goal: Show $P(k + 1)$:

Conclusion: Therefore, $P(n)$ holds for all $n \in \mathbb{Z}^+$ by the principle of induction.

Problem 9 – Review: Induction

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Let $P(n)$ be "$1 + 5 + 9 + \cdots + (4n - 3) = n(2n - 1)$". We show $P(n)$ holds for all $n \in \mathbb{Z}^+$ by induction on n.

Base Case: $P(1)$: We have $1 = 1(1) = 1(2 - 1)$ which is P(1) so the base case holds.

Inductive Hypothesis: Suppose $P(k)$ holds for an arbitrary $k \geq b$.

Inductive Step: Goal: Show $P(k+1)$:

Conclusion: Therefore, $P(n)$ holds for all $n \in \mathbb{Z}^+$ by the principle of induction.

Problem 9 – Review: Induction

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Let $P(n)$ be "$1 + 5 + 9 + \cdots + (4n - 3) = n(2n - 1)$". We show $P(n)$ holds for all $n \in \mathbb{Z}^+$ by induction on n.

<u>Base Case:</u> $P(1)$: We have $1 = 1(1) = 1(2 - 1)$ which is P(1) so the base case holds.

<u>Inductive Hypothesis:</u> Suppose $P(k)$ holds for an arbitrary $k \geq 1$. i.e. $1 + 5 + 9 + \cdots + (4k - 3) = k(2k - 1)$

<u>Inductive Step:</u> Goal: Show $P(k + 1)$:

<u>Conclusion:</u> Therefore, $P(n)$ holds for all $n \in \mathbb{Z}^+$ by the principle of induction.

Problem 9 – Review: Induction

(b) Use induction to prove that for every positive integer n, $1 + 5 + 9 + \cdots + (4n - 3) = n(2n - 1)$

Let $P(n)$ be "$1 + 5 + 9 + \cdots + (4n - 3) = n(2n - 1)$". We show $P(n)$ holds for all $n \in \mathbb{Z}^+$ by induction on n.

Base Case: $P(1)$: We have $1 = 1(1) = 1(2 - 1)$ which is P(1) so the base case holds.

Inductive Hypothesis: Suppose $P(k)$ holds for an arbitrary $k \geq 1$. i.e. $1 + 5 + 9 + \cdots + (4k - 3) = k(2k - 1)$

Inductive Step: Goal: Show $P(k + 1)$: $1 + 5 + 9 + \cdots + (4(k + 1) - 3) = (k + 1)(2(k + 1) - 1)$

Conclusion: Therefore, $P(n)$ holds for all $n \in \mathbb{Z}^+$ by the principle of induction.

Problem 9 – Review: Induction

(b) Use induction to prove that for every positive integer n, $1 + 5 + 9 + \cdots + (4n - 3) = n(2n - 1)$

Let $P(n)$ be "$1 + 5 + 9 + \cdots + (4n - 3) = n(2n - 1)$". We show $P(n)$ holds for all $n \in \mathbb{Z}^+$ by induction on n.

Base Case: $P(1)$: We have $1 = 1(1) = 1(2 - 1)$ which is P(1) so the base case holds.

Inductive Hypothesis: Suppose $P(k)$ holds for an arbitrary $k \geq 1$. i.e. $1 + 5 + 9 + \cdots + (4k - 3) = k(2k - 1)$

Inductive Step: Goal: Show $P(k+1)$: $1 + 5 + 9 + \cdots + (4(k + 1) - 3) = (k + 1)(2(k + 1) - 1)$

We have:

$1 + 5 + 9 + \cdots + (4(k + 1) - 3) = 1 + 5 + 9 + \cdots + (4k - 3) + (4(k + 1) - 3)$

$= k(2k - 1) + (4(k + 1) - 3)$ [Inductive Hypothesis]

$= k(2k - 1) + (4k + 1) = 2k\ 2 + 3k + 1 = (k + 1)(2k + 1)$ [Factor]

$= (k + 1)(2(k + 1) - 1)$

This proves P(k + 1).

Conclusion: Therefore, $P(n)$ holds for all $n \in \mathbb{Z}^+$ by the principle of induction.

Problem 10 – Review: Languages

(a) Construct a regular expression that represents binary strings where no occurrence of 11 is followed by a 0.

(b) Construct a CFG that represents the following language: $\{1^x 2^y 3^y 4^x : x, y \geq 0\}$

(c) Construct a DFA that recognizes the language of all binary strings which, when interpreted as a binary number, are divisible by 3. e.g. 11 is 3 in base-10, so should be accepted while 111 is 7 in base-10, so should be rejected. The first bit processed will be the most-significant bit.
Hint: you need to keep track of the remainder %3. What happens to a binary number when you add a 0 at the end? A 1? It's a lot like a shift operation...

(d) Construct a DFA that recognizes the language of all binary strings with an even number of 0's and each 0 is (immediately) followed by at least one 1.

Work on this problem with the people around you.

Problem 10– Review: Languages

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Problem 10 – Review: Languages

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$(0^* (10)^*)^* 1^*$

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$(0^* (10)^*)^* 1^*$

(b) Construct a CFG that represents the following language: $\{1^x 2^y 3^y 4^x : x, y \geq 0\}$

$\mathbf{S} \rightarrow 1\mathbf{S}4 \mid \mathbf{T}$

$\mathbf{T} \rightarrow 2\mathbf{T}3 \mid \varepsilon$

Problem 10– Review: Languages

(c) Construct a DFA that recognizes the language of all binary strings which, when interpreted as a binary number, are divisible by 3. e.g. 11 is 3 in base-10, so should be accepted while 111 is 7 in base-10, so should be rejected. The first bit processed will be the most-significant bit.

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Problem 10– Review: Languages

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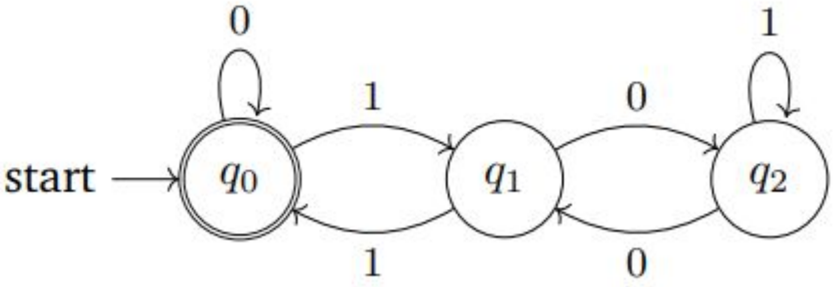

Problem 10– Review: Languages

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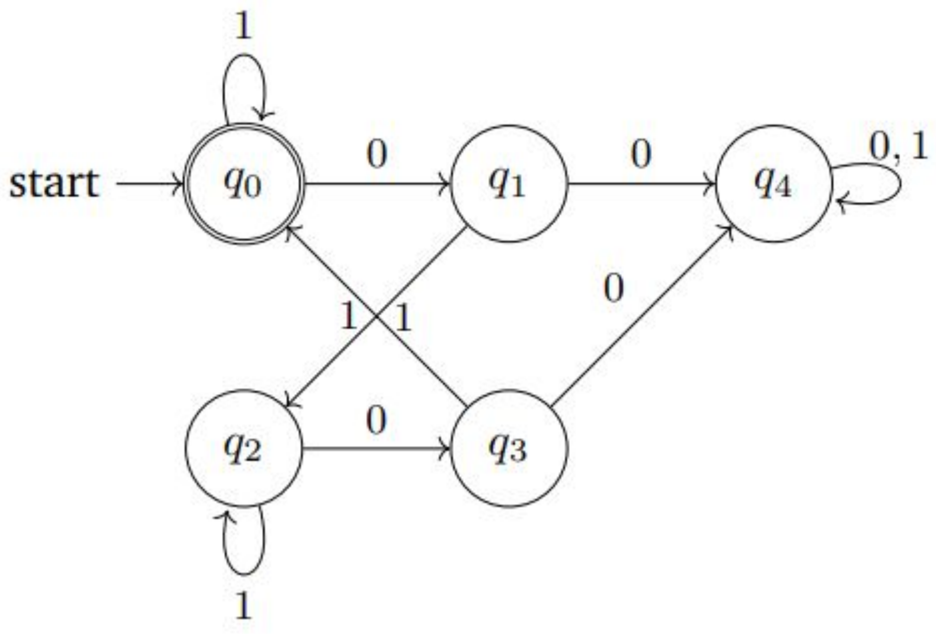

q0: even number of 0's, with final 0 followed by at least one 1

q1: odd number of 0's, with final 0 not yet followed by at least one 1

q2: odd number of 0's, with final 0 followed by at least one 1

q3: even number of 0's, with final 0 not yet followed by at least one 1

q4: garbage state where at least one 0 is not followed by at least one 1

Problem 11– Review: Uncountability

(a) Let S be the set of all real numbers in [0, 1) that only have 0s and 1s in their decimal representation. Prove that S is uncountably infinite.

Problem 11– Review: Uncountability

(a) Let S be the set of all real numbers in [0, 1) that only have 0s and 1s in their decimal representation. Prove that S is uncountably infinite.

Suppose for the sake of contradiction that S is countable. Then there exists a surjection $f : \mathbb{N} \to S$. So for each natural number i, we have some decimal sequence of 0s and 1s that i maps to.

We now construct an element x. We start x with 0. (a zero and decimal point). Then for all $i \in \mathbb{N}$, let the ith digit after the decimal point be 1 if $f(i) = 0$, and 0 if $f(i) = 1$.

Note that by our construction, for any $i \in N$, $f(i)$ differs from x on the i-th digit after the decimal point. Furthermore by our construction, x contains only 0s and 1s in its decimal expansion and $x \in [0, 1)$, so $x \in S$. Since $x \in S$ but is not in the range of f, f is not surjective. This is a contradiction. Therefore S is uncountable.

That's All, Folks!

Thanks for coming to section this week!
Any questions?