CSE 311 Section 08

Induction, Recursively Defined Sets, and One-to-One and Onto

Administrivia

Announcements & Reminders

- Midterm
  - Please don't talk about the midterm!! Not everyone has taken it yet ☺

- HW6 Regrade Requests
  - Regrade request window open as usual
  - If something was graded incorrectly, submit a regrade request

- HW7
  - Due **Friday** 11/22 @ 11:59pm (note the unusual day!)
  - Late due date Monday 11/25 @ 11:59 PM

Recursively Defined Sets

Recursive Definition of Sets

Define a set S as follows:

Basis Step:
Describe the basic starting elements in your set
ex: $0 \in S$

Recursive Step:
Describe how to derive new elements of the set from previous elements
ex: If $x \in S$ then $x + 2 \in S$.

Exclusion Rule: Every element of S is in S from the basis step (alone) or a finite number of recursive steps starting from a basis step.

Problem 1 – Recursively Defined Sets

For each of the following, write a recursive definition of the sets satisfying the following properties. Briefly justify that your solution is correct.

a) Binary strings of even length.

a) Binary strings not containing 10.

a) Binary strings not containing 10 as a substring and having at least as many 1s as 0s.

a) Binary strings containing at most two 0s and at most two 1s.

Work on this problem with the people around you.

Structural Induction

Idea of Structural Induction

Every element is built up recursively…

So to show $P(s)$ for all $s \in S$…

Show $P(b)$ for all base case elements b.

Show for an arbitrary element not in the base case, if $P()$ holds for every named element in the recursive rule, then $P()$ holds for the new element (each recursive rule will be a case of this proof).

Structural Induction Template

Let $P(x)$ be "<predicate>". We show $P(x)$ holds for all $x \in S$ by structural induction.

Base Case: Show $P(x)$
[Do that for every base cases x in S.]

Inductive Hypothesis: Suppose $P(x)$ for an arbitrary x
[Do that for every x listed as in S in the recursive rules.]

Inductive Step: Show $P()$ holds for y.
[You will need a separate case/step for every recursive rule.]

Therefore $P(x)$ holds for all $x \in S$ by the principle of induction.

Problem 2b – Structural Induction on Trees

Definition of Tree:
Basis Step: • is a Tree.
Recursive Step: If L is a Tree and R is a Tree then Tree(•, L, R) is a Tree

Definition of leaves():
leaves(•) = 1
leaves(Tree(•, L, R)) = leaves(L) + leaves(R)

Definition of size():
size(•) = 1
size(Tree(•, L, R)) =1 + size(L) + size(R)

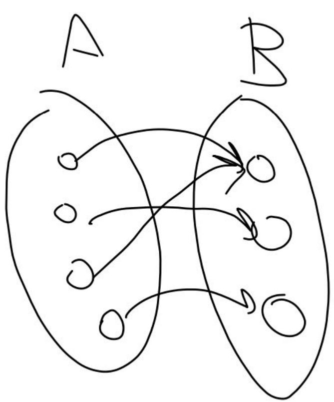
Prove that leaves(T) ≥ size(T)/2 + 1/2 for all Trees T

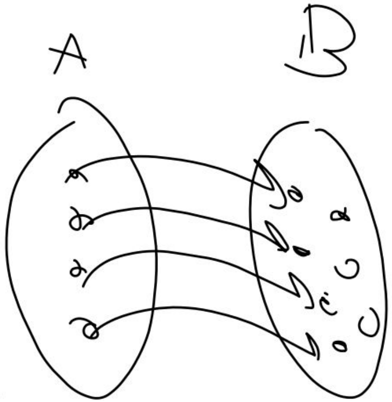
Work on this problem with the people around you.

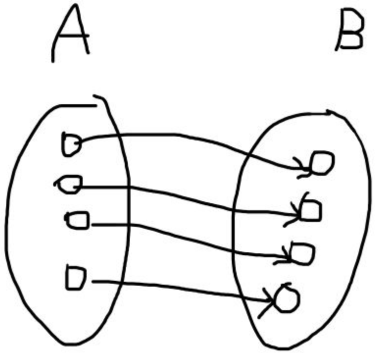
One-to-One and Onto

Review of function definitions

Match the definition to the description! (One to one, onto or bijective?)



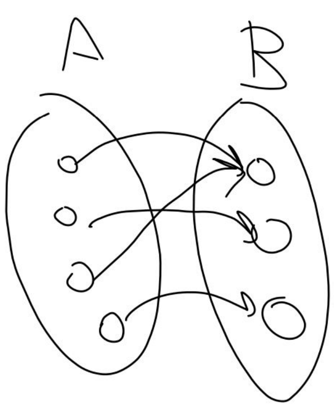


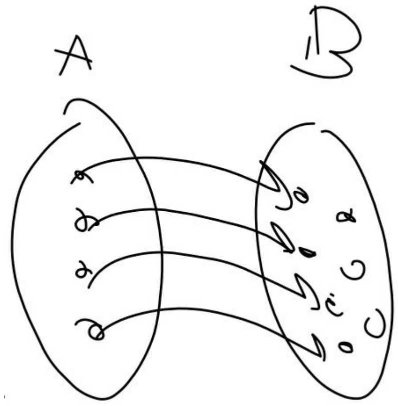


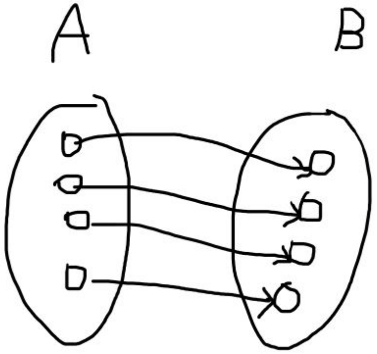
Review of function definitions

Match the definition to the description! (One to one, onto or bijective?)

Onto function
Every element of the codomain has <u>at least one element</u> in the domain mapping to it





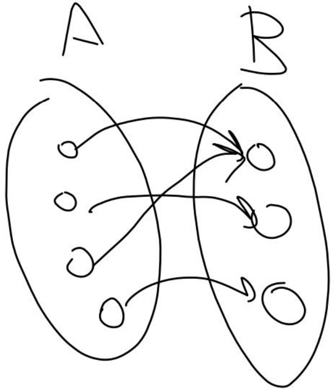


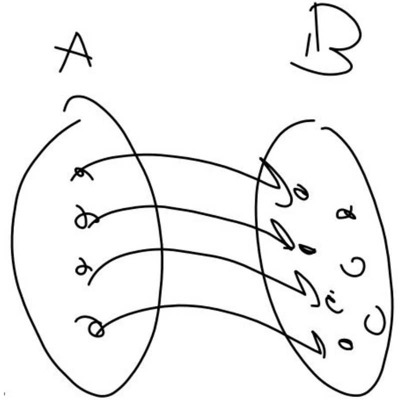
Review of function definitions

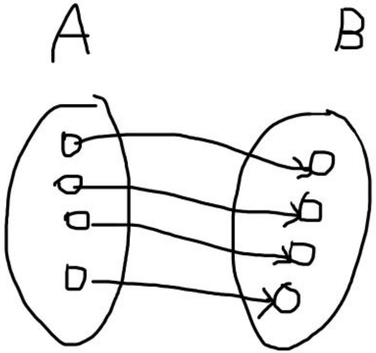
Match the definition to the description! (One to one, onto or bijective?)

Onto function
Every element of the codomain has at least one element in the domain mapping to it

One to one function
Every element of the codomain has at most one element in the domain mapping to it



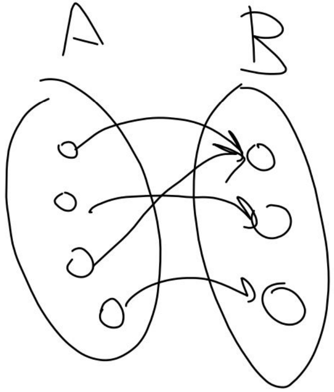




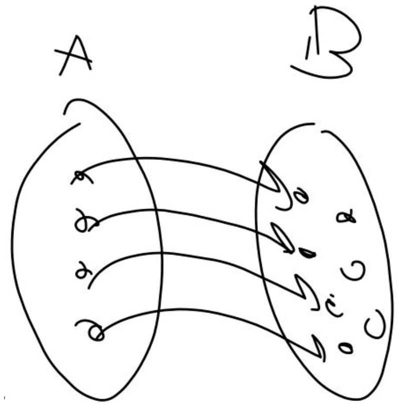
Review of function definitions

Match the definition to the description! (One to one, onto or bijective?)

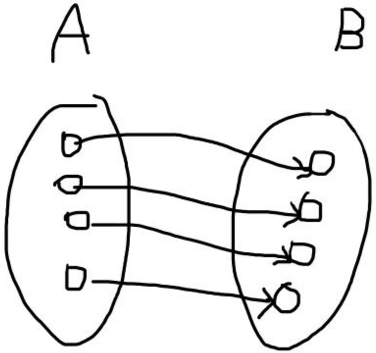
Onto function
Every element of the codomain has <u>at least one element</u> in the domain mapping to it



One to one function
Every element of the codomain has <u>at most one element</u> in the domain mapping to it



Bijection
Every element of the codomain has <u>exactly one element</u> in the domain mapping to it



Problem 8 – One-to-One and Onto

For each of these functions, state whether it is one-to-one, onto, both, or neither.

$a)\ f : \mathbb{N} \to \mathbb{N}, f(x) = x^2$

$b)\ f : \mathbb{R} \to \mathbb{R}, f(x) = x^2$

$c)\ f : \mathbb{R}^+ \to \mathbb{R}^+, f(x) = x^2$

That's All, Folks!

Thanks for coming to section this week!
Any questions?