CSE 311 Section 2

Logic and Equivalences

Administrivia & Introductions

Announcements & Reminders

- Sections are Graded
  - You will be graded on section participation, so please try to come ☺

- HW1 due YESTERDAY on Gradescope
  - Remember, you have 6 late days to use throughout the quarter
  - You can use up to 3 late days on any 1 assignment
  - You don’t get extra credit for having any unused late days, so feel free to use them if you need them!

- Check the course website for OH times!
  - There are office hours every day, so come visit if you have questions!

References

- Helpful reference sheets can be found on the course website!
  - https://courses.cs.washington.edu/courses/cse311/24au/resources/
- How to LaTeX (found on Assignments page of website):
  - https://courses.cs.washington.edu/courses/cse311/24au/assignments/HowToLaTeX.pdf
- Equivalence Reference Sheet
  - https://courses.cs.washington.edu/courses/cse311/24au/resources/reference-logical_equiv.pdf
  - https://courses.cs.washington.edu/courses/cse311/24au/resources/logicalConnectPoster.pdf
- Boolean Algebra Reference Sheet
  - https://courses.cs.washington.edu/courses/cse311/24au/resources/reference-boolean-alg.pdf
- Plus more!

Typesetting

- You are STRONGLY ENCOURAGED to use LaTeX for your assignments.
- We have lots of resources available to help you get started typesetting with LaTeX!
- Come to office hours and we are happy to answer any questions!!

```latex
\begin{align*}
  \neg p \rightarrow (q \rightarrow r) &\equiv \neg \neg p \vee (q \rightarrow r) &&\text{Law Of Impl.} \\
  &\equiv p \vee (q \rightarrow r) &&\text{Double Neg} \\
  &\equiv p \vee (\neg q \vee r) &&\text{Law of Impl.} \\
  &\equiv (p \vee \neg q) \vee r &&\text{Assoc.} \\
  &\equiv (\neg q \vee p) \vee r &&\text{Comm.} \\
  &\equiv \neg q \vee (p \vee r) &&\text{Assoc.} \\
  &\equiv q \rightarrow (p \vee r) &&\text{Law of Imp.}
\end{align*}
```

$$\begin{align*}
\neg p \rightarrow (q \rightarrow r) &\equiv \neg \neg p \vee (q \rightarrow r) &&\text{Law of Impl.} \\
&\equiv p \vee (q \rightarrow r) &&\text{Double Neg} \\
&\equiv p \vee (\neg q \vee r) &&\text{Law of Impl.} \\
&\equiv (p \vee \neg q) \vee r &&\text{Assoc.} \\
&\equiv (\neg q \vee p) \vee r &&\text{Comm.} \\
&\equiv \neg q \vee (p \vee r) &&\text{Assoc.} \\
&\equiv q \rightarrow (p \vee r) &&\text{Law of Imp.}
\end{align*}$$

Symbolic Proof

Equivalence Proof Review

$p \wedge (p \rightarrow q) \equiv p \wedge (\neg p \vee q)$	[Law of Implication]
$\equiv (p \wedge \neg p) \vee (p \wedge q)$	[Distributivity]
$\equiv F \vee (p \wedge q)$	[Negation]
$\equiv (p \wedge q) \vee F$	[Commutativity]
$\equiv p \wedge q$	[Identity]

Notice: You may be tempted to use the **Identity** property immediately if you see **F ∨ p** but you need to use **Commutativity** first!

Problem 4 – Equivalences

Prove that each of the following pairs of propositional formulae are equivalent using propositional equivalences.

a) $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$
You may use the rule: $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

b) $\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r)$

Work on part (b) with the people around you, and then we'll go over it together!

Problem 4 – Equivalences

b) $\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r)$

Problem 4 – Equivalences

b) $\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r)$

$\neg p \rightarrow (q \rightarrow r) \equiv \ldots$

$\equiv q \rightarrow (p \vee r)$

Problem 4 – Equivalences

b) $\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r)$

$\neg p \rightarrow (q \rightarrow r) \equiv \neg\neg p \vee (q \rightarrow r)$ [Law of Implication]

$\equiv \neg q \vee (p \vee r)$

$\equiv q \rightarrow (p \vee r)$ [Law of Implication]

Problem 4 – Equivalences

b) $\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r)$

$\neg p \rightarrow (q \rightarrow r) \equiv \neg\neg p \vee (q \rightarrow r)$ [Law of Implication]

$\equiv p \vee (q \rightarrow r)$ [Double Negation]

$\equiv \neg q \vee (p \vee r)$

$\equiv q \rightarrow (p \vee r)$ [Law of Implication]

Problem 4 – Equivalences

b) $\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r)$

$$
\begin{aligned}
\neg p \rightarrow (q \rightarrow r) &\equiv \neg\neg p \vee (q \rightarrow r) && \text{[Law of Implication]} \\
&\equiv p \vee (q \rightarrow r) && \text{[Double Negation]} \\
&\equiv p \vee (\neg q \vee r) && \text{[Law of Implication]} \\
&\equiv \neg q \vee (p \vee r) && \\
&\equiv q \rightarrow (p \vee r) && \text{[Law of Implication]}
\end{aligned}
$$

Problem 4 – Equivalences

b) $\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r)$

$\neg p \rightarrow (q \rightarrow r) \equiv \neg\neg p \vee (q \rightarrow r)$	[Law of Implication]
$\equiv p \vee (q \rightarrow r)$	[Double Negation]
$\equiv p \vee (\neg q \vee r)$	[Law of Implication]
$\equiv (p \vee \neg q) \vee r$	[Associativity]
$\equiv \neg q \vee (p \vee r)$	
$\equiv q \rightarrow (p \vee r)$	[Law of Implication]

Problem 4 – Equivalences

b) $\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r)$

$\neg p \rightarrow (q \rightarrow r) \equiv \neg\neg p \vee (q \rightarrow r)$	[Law of Implication]
$\equiv p \vee (q \rightarrow r)$	[Double Negation]
$\equiv p \vee (\neg q \vee r)$	[Law of Implication]
$\equiv (p \vee \neg q) \vee r$	[Associativity]
$\equiv (\neg q \vee p) \vee r$	[Commutativity]
$\equiv \neg q \vee (p \vee r)$	
$\equiv q \rightarrow (p \vee r)$	[Law of Implication]

Problem 4 – Equivalences

b) $\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r)$

$\neg p \rightarrow (q \rightarrow r) \equiv \neg\neg p \vee (q \rightarrow r)$	[Law of Implication]
$\equiv p \vee (q \rightarrow r)$	[Double Negation]
$\equiv p \vee (\neg q \vee r)$	[Law of Implication]
$\equiv (p \vee \neg q) \vee r$	[Associativity]
$\equiv (\neg q \vee p) \vee r$	[Commutativity]
$\equiv \neg q \vee (p \vee r)$	[Associativity]
$\equiv q \rightarrow (p \vee r)$	[Law of Implication]

Problem 4 – Equivalences

a) $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$

You may use the rule: $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$	[iff is two implications]
$\equiv (\neg p \vee q) \wedge (q \rightarrow p)$	[Law of Implication]
$\equiv (\neg p \vee q) \wedge (\neg q \vee p)$	[Law of Implication]
$\equiv ((\neg p \vee q) \wedge \neg q) \vee ((\neg p \vee q) \wedge p)$	[Distributivity]
$\equiv (\neg q \wedge (\neg p \vee q)) \vee ((\neg p \vee q) \wedge p)$	[Commutativity]
$\equiv ((\neg q \wedge \neg p) \vee (\neg q \wedge q)) \vee ((\neg p \vee q) \wedge p)$	[Distributivity]
$\equiv ((\neg q \wedge \neg p) \vee (q \wedge \neg q)) \vee ((\neg p \vee q) \wedge p)$	[Commutativity]
$\equiv ((\neg q \wedge \neg p) \vee F) \vee ((\neg p \vee q) \wedge p)$	[Negation]
$\equiv (\neg q \wedge \neg p) \vee ((\neg p \vee q) \wedge p)$	[Identity]
$\equiv (\neg p \wedge \neg q) \vee ((\neg p \vee q) \wedge p)$	[Commutativity]
$\equiv (\neg p \wedge \neg q) \vee (p \wedge (\neg p \vee q))$	[Commutativity]
$\equiv (\neg p \wedge \neg q) \vee ((p \wedge \neg p) \vee (p \wedge q))$	[Distributivity]
$\equiv (\neg p \wedge \neg q) \vee (F \vee (p \wedge q))$	[Negation]
$\equiv (\neg p \wedge \neg q) \vee ((p \wedge q) \vee F)$	[Commutativity]
$\equiv (\neg p \wedge \neg q) \vee (p \wedge q)$	[Identity]
$\equiv (p \wedge q) \vee (\neg p \wedge \neg q)$	[Commutativity]

Normal Forms

(Canonical) Normal Forms

- Standard ways of translating a truth table into a proposition.
- We already did these in lecture when we translated implications into an expression only using ands, ors, and nots!
- Once you translate into one of these forms, **don’t simplify your expression any further!** It often looks like you can factor variables out to make it prettier, but the whole point is to write the expression into this standardized way, so just leave it as-is ☺

DNF (OR of ANDs)

- Disjunctive Normal Form
  - OR of ANDs
  - Method:
    1. Read the TRUE rows of the truth table
    2. AND together all the variable settings in a given (true) row
    3. OR together the true rows

DNF (OR of ANDs)

1. Read the TRUE rows of the truth table
2. AND together all the variable settings in a given (true) row
3. OR together the true rows

p	*q*	*G(p,q)*	
T	T	T	$p \land q$
T	F	F	
F	T	T	$\neg p \land q$
F	F	F	

$$G(p,q) \equiv (p \land q) \lor (\neg p \land q)$$

CNF (AND of ORs)

- Conjunctive Normal Form
  - AND of ORs
  - Method:
    1. Read the FALSE rows of the truth table
    2. OR together the negations of all the variable settings in the false row
    3. AND together the false rows

CNF (AND of ORs)

1. Read the FALSE rows of the truth table
2. OR together the negations of all the variable settings in the false row
3. AND together the false rows

p	*q*	*G(p,q)*	
T	T	T	
T	F	F	$\neg p \lor q$
F	T	T	
F	F	F	$p \lor q$

$$G(p, q) \equiv (\neg p \lor q) \land (p \lor q)$$

Problem 6 – Canonical Forms

Consider the functions $F(A, B, C)$ and $G(A, B, C)$ specified by the following truth table:

a) Write the DNF and CNF expressions for $F(A, B, C)$.

b) Write the DNF and CNF expressions for $G(A, B, C)$.

A	B	C	F(A,B,C)	G(A,B,C)
T	T	T	T	F
T	T	F	T	T
T	F	T	F	F
T	F	F	F	F
F	T	T	T	T
F	T	F	T	F
F	F	T	F	T
F	F	F	T	F

Problem 6 – Canonical Forms

a) Write the DNF and CNF expressions for $F(A, B, C)$.

b) Write the DNF and CNF expressions for $G(A, B, C)$.

A	B	C	F(A,B,C)	G(A,B,C)
T	T	T	T	F
T	T	F	T	T
T	F	T	F	F
T	F	F	F	F
F	T	T	T	T
F	T	F	T	F
F	F	T	F	T
F	F	F	T	F

Work on part (a) with the people around you, and then we'll go over it together!

Problem 6 – Canonical Forms

a) Write the DNF and CNF expressions for $F(A, B, C)$.

A	*B*	*C*	*F(A,B,C)*	*G(A,B,C)*
T	T	T	T	F
T	T	F	T	T
T	F	T	F	F
T	F	F	F	F
F	T	T	T	T
F	T	F	T	F
F	F	T	F	T
F	F	F	T	F

Problem 6 – Canonical Forms

a) Write the DNF and CNF expressions for $F(A,B,C)$.

A	B	C	F(A,B,C)	G(A,B,C)
T	T	T	T	F
T	T	F	T	T
T	F	T	F	F
T	F	F	F	F
F	T	T	T	T
F	T	F	T	F
F	F	T	F	T
F	F	F	T	F

DNF: (OR of ANDs)

Problem 6 – Canonical Forms

a) Write the DNF and CNF expressions for $F(A, B, C)$.

A	B	C	F(A,B,C)	G(A,B,C)
T	T	T	T	F
T	T	F	T	T
T	F	T	F	F
T	F	F	F	F
F	T	T	T	T
F	T	F	T	F
F	F	T	F	T
F	F	F	T	F

DNF: (OR of ANDs)

Problem 6 – Canonical Forms

a) Write the DNF and CNF expressions for $F(A,B,C)$.

DNF: (OR of ANDs)

	A	B	C	F(A,B,C)	G(A,B,C)
$A \wedge B \wedge C$	T	T	T	T	F
$A \wedge B \wedge \neg C$	T	T	F	T	T
	T	F	T	F	F
	T	F	F	F	F
$\neg A \wedge B \wedge C$	F	T	T	T	T
$\neg A \wedge B \wedge \neg C$	F	T	F	T	F
	F	F	T	F	T
$\neg A \wedge \neg B \wedge \neg C$	F	F	F	T	F

Problem 6 – Canonical Forms

a) Write the DNF and CNF expressions for $F(A, B, C)$.

DNF: (OR of ANDs)

	A	B	C	F(A,B,C)	G(A,B,C)
$A \wedge B \wedge C$	T	T	T	T	F
$A \wedge B \wedge \neg C$	T	T	F	T	T
	T	F	T	F	F
	T	F	F	F	F
$\neg A \wedge B \wedge C$	F	T	T	T	T
$\neg A \wedge B \wedge \neg C$	F	T	F	T	F
	F	F	T	F	T
$\neg A \wedge \neg B \wedge \neg C$	F	F	F	T	F

$$(\boldsymbol{A \wedge B \wedge C}) \vee (\boldsymbol{A \wedge B \wedge \neg C}) \vee (\boldsymbol{\neg A \wedge B \wedge C}) \vee (\boldsymbol{\neg A \wedge B \wedge \neg C}) \vee (\boldsymbol{\neg A \wedge \neg B \wedge \neg C})$$

Problem 6 – Canonical Forms

a) Write the DNF and CNF expressions for $F(A,B,C)$.

A	*B*	*C*	*F(A,B,C)*	*G(A,B,C)*
T	T	T	T	F
T	T	F	T	T
T	F	T	F	F
T	F	F	F	F
F	T	T	T	T
F	T	F	T	F
F	F	T	F	T
F	F	F	T	F

CNF: (AND of ORs)

Problem 6 – Canonical Forms

a) Write the DNF and CNF expressions for $F(A, B, C)$.

CNF: (AND of ORs)

A	B	C	F(A,B,C)	G(A,B,C)
T	T	T	T	F
T	T	F	T	T
T	F	T	F	F
T	F	F	F	F
F	T	T	T	T
F	T	F	T	F
F	F	T	F	T
F	F	F	T	F

Problem 6 – Canonical Forms

a) Write the DNF and CNF expressions for $F(A, B, C)$.

CNF: (AND of ORs)

	A	B	C	F(A,B,C)	G(A,B,C)
	T	T	T	T	F
	T	T	F	T	T
$\neg A \vee B \vee \neg C$	T	F	T	F	F
$\neg A \vee B \vee C$	T	F	F	F	F
	F	T	T	T	T
	F	T	F	T	F
$A \vee B \vee \neg C$	F	F	T	F	T
	F	F	F	T	F

Problem 6 – Canonical Forms

a) Write the DNF and CNF expressions for $F(A, B, C)$.

CNF: (AND of ORs)

A	B	C	F(A,B,C)	G(A,B,C)
T	T	T	T	F
T	T	F	T	T
T	F	T	F	F
T	F	F	F	F
F	T	T	T	T
F	T	F	T	F
F	F	T	F	T
F	F	F	T	F

Row T F T: $\neg A \vee B \vee \neg C$

Row T F F: $\neg A \vee B \vee C$

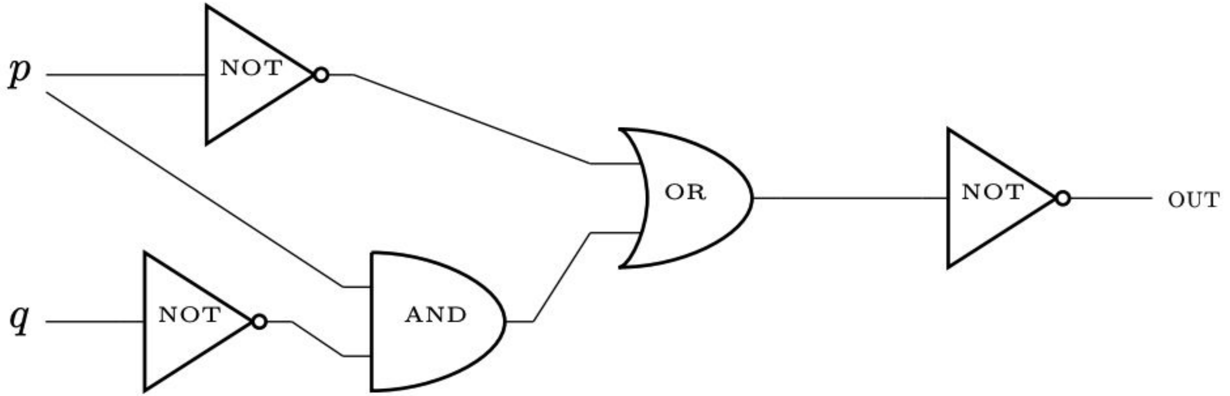
Row F F T: $A \vee B \vee \neg C$

$$(\neg \boldsymbol{A} \vee \boldsymbol{B} \vee \neg \boldsymbol{C}) \wedge (\neg \boldsymbol{A} \vee \boldsymbol{B} \vee \boldsymbol{C}) \wedge (\boldsymbol{A} \vee \boldsymbol{B} \vee \neg \boldsymbol{C})$$

Circuits

Problem 3 – Circuitous

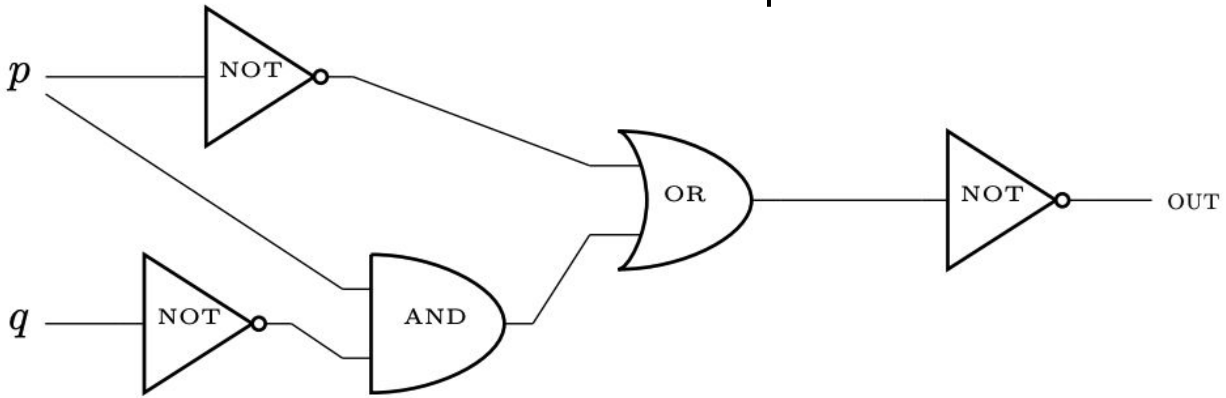
Translate the following circuit into a logical expression.



Work on this problem with the people around you, and then we'll go over it together!

Problem 3 – Circuitous

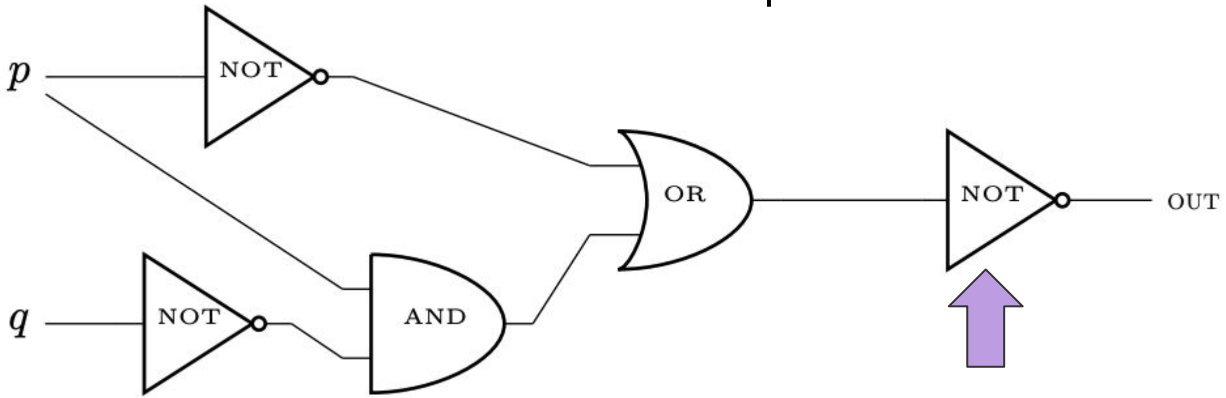
Translate the following circuit into a logical expression.



Tip: Think of starting from the end and working back to create the expression.

Problem 3 – Circuitous

Translate the following circuit into a logical expression.

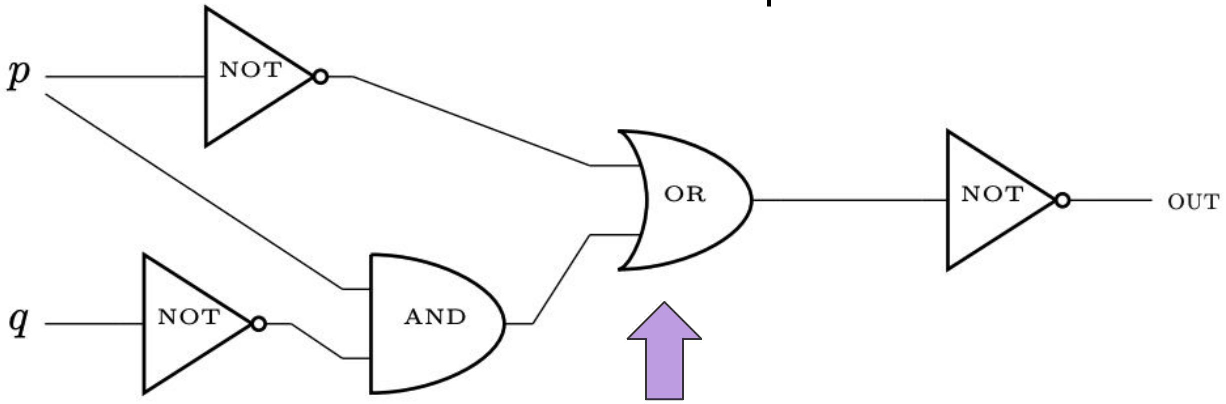


Tip: Think of starting from the end and working back to create the expression.

$\neg(\ldots)$

Problem 3 – Circuitous

Translate the following circuit into a logical expression.

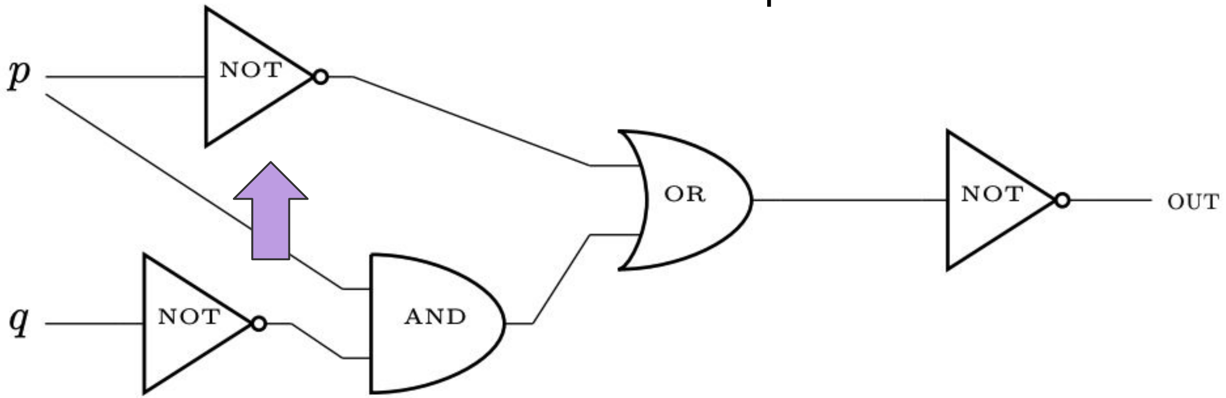


Tip: Think of starting from the end and working back to create the expression.

¬(… ∨ …)

Problem 3 – Circuitous

Translate the following circuit into a logical expression.

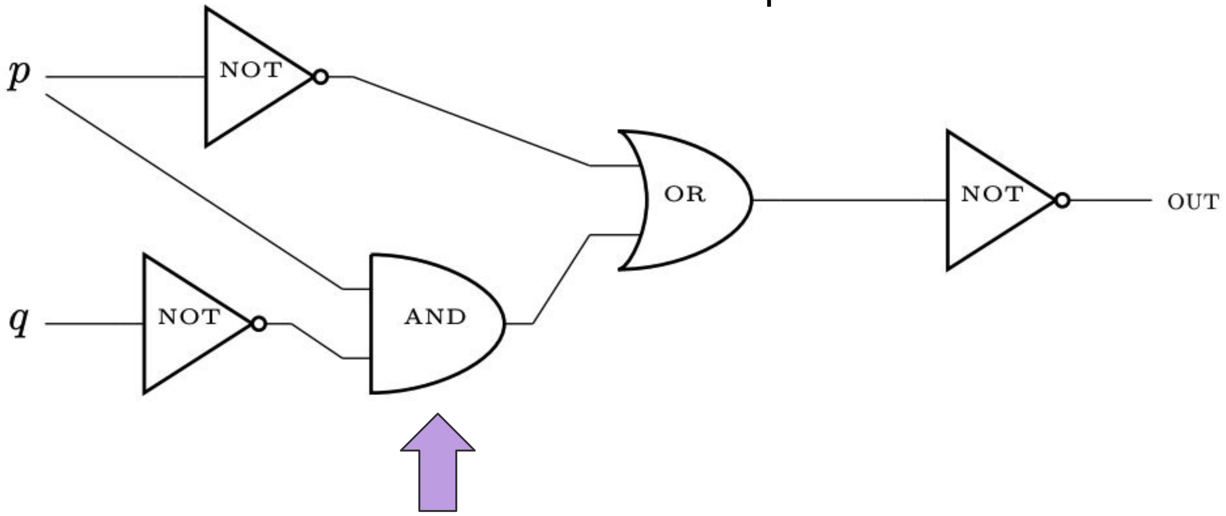


Tip: Think of starting from the end and working back to create the expression.

¬(¬ ... ∨ ...)

Problem 3 – Circuitous

Translate the following circuit into a logical expression.

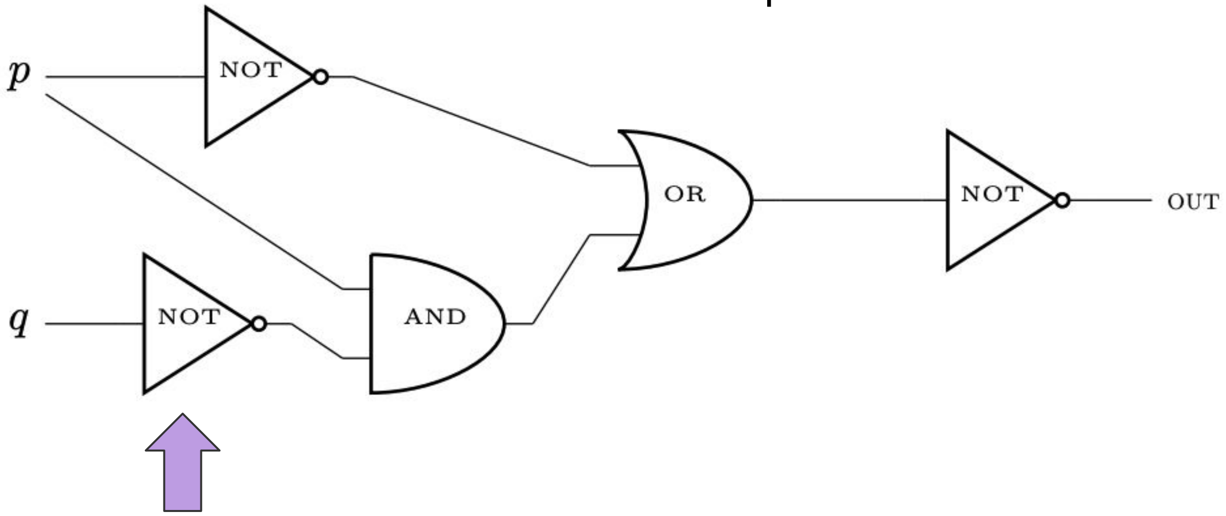


Tip: Think of starting from the end and working back to create the expression.

¬(¬ … ∨ (… ∧ …))

Problem 3 – Circuitous

Translate the following circuit into a logical expression.

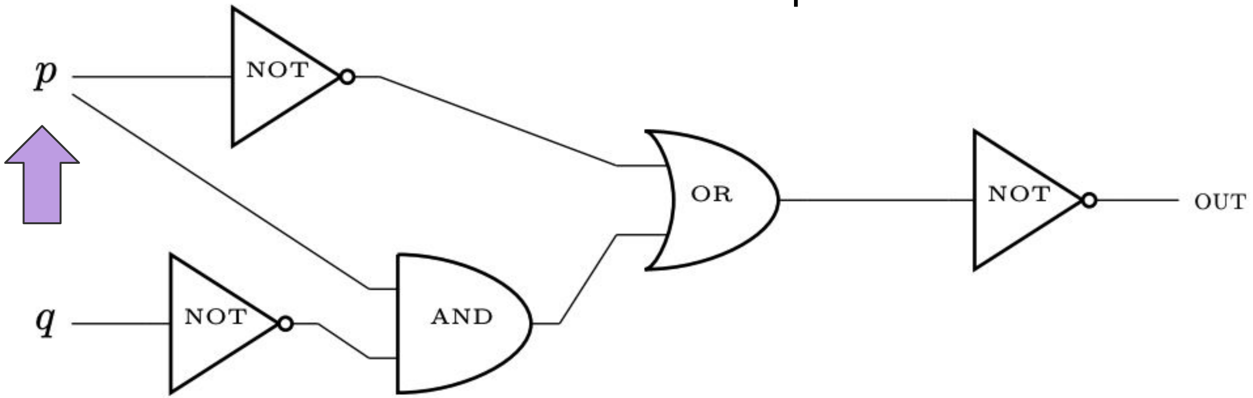


Tip: Think of starting from the end and working back to create the expression.

$\neg(\neg \ldots \lor (\ldots \land \neg \ldots))$

Problem 3 – Circuitous

Translate the following circuit into a logical expression.

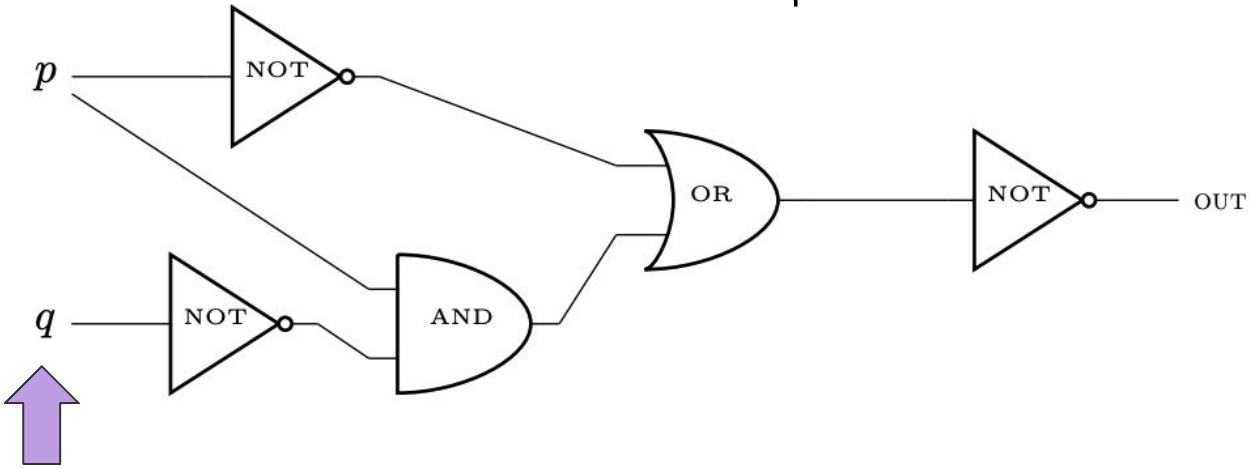


Tip: Think of starting from the end and working back to create the expression.

$\neg(\neg p \vee (p \wedge \neg \ldots))$

Problem 3 – Circuitous

Translate the following circuit into a logical expression.



Tip: Think of starting from the end and working back to create the expression.

$\neg(\neg p \lor (p \land \neg q))$

Boolean Algebra

Boolean Algebra Review

Boolean algebra is another way of representing logic symbolically. Remember, it is equivalent to the symbolic logic we have already learned, it just uses different symbols!

- "And" represented by: ● (multiplication)
- "Or" represented by: + (addition)
- "Not" represented by: ′ (apostrophe after the variable)

Problem 5 – Boolean Algebra

For each of the following parts, write the logical expression using boolean algebra operators. Then, simplify it using axioms and theorems of boolean algebra.

a) $\neg p \vee (\neg q \vee (p \wedge q))$

b) $\neg(p \vee (q \wedge p))$

Work on part (a) with the people around you, and then we'll go over it together!

Problem 5 – Boolean Algebra

a) $\neg p \vee (\neg q \vee (p \wedge q))$

Problem 5 – Boolean Algebra

a) $\neg p \vee (\neg q \vee (p \wedge q))$

$$p' + q' + pq$$

Problem 5 – Boolean Algebra

a) $\neg p \vee (\neg q \vee (p \wedge q))$

$p' + q' + pq = (pq)' + pq$ [DeMorgan's]

Problem 5 – Boolean Algebra

a) $\neg p \lor (\neg q \lor (p \land q))$

$$p' + q' + pq = (pq)' + pq \qquad \text{[DeMorgan's]}$$

$$= pq + (pq)' \qquad \text{[Commutativity]}$$

Problem 5 – Boolean Algebra

a) $\neg p \vee (\neg q \vee (p \wedge q))$

$$
\begin{aligned}
p' + q' + pq &= (pq)' + pq && \text{[DeMorgan's]} \\
&= pq + (pq)' && \text{[Commutativity]} \\
&= 1 && \text{[Complementarity]}
\end{aligned}
$$

Remember that 1 in Boolean Algebra is equivalent to T in propositional logic.
When something always evaluates to true, it is a **tautology**!

Problem 5 – Boolean Algebra

a) $\neg p \vee (\neg q \vee (p \wedge q))$

We can also double check our answer by simplifying the propositional logic:

$\neg p \vee (\neg q \vee (p \wedge q) \equiv \neg p \vee ((\neg q \vee p) \wedge (\neg q \vee q))$	[Distributivity]
$\equiv \neg p \vee ((\neg q \vee p) \wedge (q \vee \neg q))$	[Commutativity]
$\equiv \neg p \vee ((\neg q \vee p) \wedge T)$	[Negation]
$\equiv \neg p \vee (\neg q \vee p)$	[Identity]
$\equiv (\neg q \vee p) \vee \neg p$	[Commutativity]
$\equiv \neg q \vee (p \vee \neg p)$	[Associativity]
$\equiv \neg q \vee T$	[Negation]
$\equiv T$	[Domination]

Just like the Boolean Algebra expression simplified to 1, this also simplified to T! No matter which notation we use, we end up with the same result. Yay!

Problem 5 – Boolean Algebra

b) $\neg(p \vee (q \wedge p))$

$$
\begin{aligned}
(p + qp)' &\equiv p'(qp)' && \text{[DeMorgan's]} \\
&\equiv p'(q' + p') && \text{[DeMorgan's]} \\
&\equiv p'(p' + q') && \text{[Commutativity]} \\
&\equiv p' && \text{[Absorption]}
\end{aligned}
$$

That’s All, Folks!

Thanks for coming to section this week!
Any questions?