

Section 01: Solutions

1. Warm-Up

Translate the English sentences below into symbolic logic.

- (a) If I am lifting weights this afternoon, then I do a warm-up exercise.

Solution:

Since we're in "if...then..." form, the sentence is an implication.

p : I am lifting weights

q : I do a warm-up exercise

$$p \rightarrow q$$

- (b) If I am cold and going to bed or I am two-years old, then I carry a blanket. **Solution:**

p : I am cold

q : I am going to bed

r : I am two-years old

s : I carry a blanket

$$[(p \wedge q) \vee r] \rightarrow s$$

How did we know the translation wasn't $[p \wedge (q \vee r)] \rightarrow s$? Two hints were available: first, omitted words ("going to bed" instead of "I am going to bed" indicates q should be closer to the "and" than the "or"), second the interpretation of the sentence – two-year olds more commonly carry blankets during the day than warm adults.

2. If I can translate, then...

For each of the following more obscure English ways to write an implication, define atomic propositions and write a symbolic representation of the sentence.

- (a) whenever I walk my dog, I make new friends. **Solution:**

p : I walk my dog

q : I make new friends

$$p \rightarrow q$$

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The promise is that we will definitely make new friends on the condition of walking our dog.

- (b) I will drink coffee, if Starbucks is open or my coffeemaker works.

Solution:

p : I will drink coffee

q : Starbucks is open

r : my coffeemaker works

$$(q \vee r) \rightarrow p$$

- (c) Being a U.S. citizen and over 18 is sufficient to be eligible to vote. **Solution:**

p : One is a U.S. citizen q : One is over 18 r : One is eligible to vote

$$(p \wedge q) \rightarrow r$$

The original sentence omits a subject. We introduced a dummy subject “one” to the propositions, you might have said “someone” or “a person” instead (among other options).

- (d) I can go home only if I have finished my homework. **Solution:**

p : I can go home. q : I have finished my homework.

$$p \rightarrow q$$

The promise here is that if I can go home then I must have finished my homework. It can sometimes help to imagine when the sentence is broken. Is it broken if my homework is finished, but I cannot go home? No, perhaps I also have to say bye to my friends before I leave. But if I can go home with unfinished homework, then the promise is broken.

“Only if” is one of the more confusing arrangements – the consequence (“the then part”) is adjacent to the “only if.”

- (e) Having an internet connection is necessary to log onto zoom. **Solution:**

p : One has an internet connection

q : One can log onto zoom

$$q \rightarrow p$$

The internet connection is not enough (what if you don’t have the meeting link?) but certainly if you are in the meeting then you have a connection.

- (f) I am a student because I attend university. **Solution:**

p : I am a student

q : I attend university

$$q \rightarrow p$$

This can be understood that *since* I attend university, then necessarily I am a student. You could also be a student if you do not attend a university, for example if you attend high school, but necessarily if you attend university then you must be a student.

3. I can rewrite these formulas in English, only if...

Given propositions and a logical formula, write **two** potential English translations. The meanings of the sentences will be the same (They represent the same formula!), but they can still look quite different.

- (a) p : The sun is out
 q : We have class outside

$$p \rightarrow q$$

Solution:

If the sun is out, then we have class outside.
Whenever the sun is out, we have class outside.

- (b) p : the book has been out for a week.
 q : I don't have homework.
 r : I have finished reading the book.

$$(p \wedge q) \rightarrow r$$

Solution:

I have finished reading the book, if it has been out for a week and I don't have homework.
The book being out for a week and me not having homework is sufficient for me to have finished reading the book.

- (c) p : I have read the manual
 q : I operate the machine

$$q \rightarrow p$$

Solution:

I operate the machine only if I have read the manual.
Operating the machine implies that I have read the manual.

4. Translation

For each of the following, define propositional variables and translate the sentences into logical notation.

- (a) I will remember to send you the address only if you send me an e-mail message. **Solution:**

p : I will remember to send you the address

q : You send me an e-mail message

$$p \rightarrow q$$

- (b) If berries are ripe along the trail, hiking is safe if and only if grizzly bears have not been seen in the area.

Solution:

p : Berries are ripe along the trail

q : Hiking is safe

r : Grizzly bears have not been seen in the area

$$p \rightarrow (q \leftrightarrow r)$$

- (c) Unless I am trying to type something, my cat is either eating or sleeping. **Solution:**

p : My cat is eating
 q : My cat is sleeping
 r : I'm trying to type

$$\neg r \rightarrow (p \oplus q)$$

5. Tea Time

Consider the following sentence:

If I am drinking tea then I am eating a cookie, or, if I am eating a cookie then I am drinking tea.

- (a) Define propositional variables and translate the sentence into an expression in logical notation. **Solution:**

p : I am drinking tea
 q : I am eating a cookie

$$(p \rightarrow q) \vee (q \rightarrow p)$$

- (b) Fill out a truth table for your expression. **Solution:**

p	q	$(p \rightarrow q)$	$(q \rightarrow p)$	$(p \rightarrow q) \vee (q \rightarrow p)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

6. Exclusive Or

Exclusive or ($\oplus$) and inclusive or ($\vee$) both can be translated as “or” in English. For each of the following ambiguous phrases, decide which type of “or” is likely meant and why.

- (a) Experience with C or Java is required. **Solution:**

Inclusive or. Experience with both is usually not a bad thing.

(b) Lunch includes soup or salad. **Solution:**

Exclusive or. Most restaurants charge you more for both.

(c) Publish or perish. **Solution:**

This phrase is a common one among researchers – it means publish papers or your career will perish. Exclusive or is meant; i.e. if you do indeed publish you should avoid the loss of your career.

(d) To enter the country, you need a passport or voter registration card. **Solution:**

Inclusive or – if you have both, they won't kick you out!