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## Full outline

1. Suppose for the sake of contradiction that L is regular. Then there is some DFA M that recognizes L.

2. Let *S* be [fill in with an infinite set of prefixes].

3. Because the DFA is finite and S is infinite, there are two (different) strings x, y in S such that x and y go to the same state when read by M [you don't get to control x, y other than having them not equal and in S]

4. Consider the string z [argue exactly one of xz, yz will be in L]

5. Since x, y both end up in the same state, and we appended the same z, both xz and yz end up in the same state of M. Since  $xz \in L$  and  $yz \notin L$ , M does not recognize L. But that's a contradiction!

6. So *L* must be an irregular language.

## Practical Tips

When you're choosing the set S, think about what the DFA would "have to count"

That is fundamentally why a language is irregular. The set S is the way we prove it! Whatever we "need to remember" it's different for every element of S.

If your strings have an "obvious middle" (like between the 0's and 1's) that's a good place to start.

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## One more, just the key steps

What about  $\{a^k b^k c^k : k \ge 0\}$ ?