One-to-one proofs

One-to-one (aka injection)

A function f is one-to-one iff
$\forall a \forall b(f(a) = f(b) \rightarrow a = b)$

It's a for-all statement! We know how to prove it.

Let $f: \mathbb{Z} \rightarrow \mathbb{Z}$ be the function given by $f(x) = x + 5$.

Claim: f is one-to-one

Proof:

Onto proofs

Onto (aka surjection)

A function $f: A \rightarrow B$ is onto iff
$\forall b \in B \exists a \in A(b = f(a))$

It's a for-all statement, with an exists inside! We know how to prove it.

Let $f: \mathbb{Z} \rightarrow \mathbb{Z}$ be the function given by $f(x) = x + 5$.

Claim: f is onto

Proof:

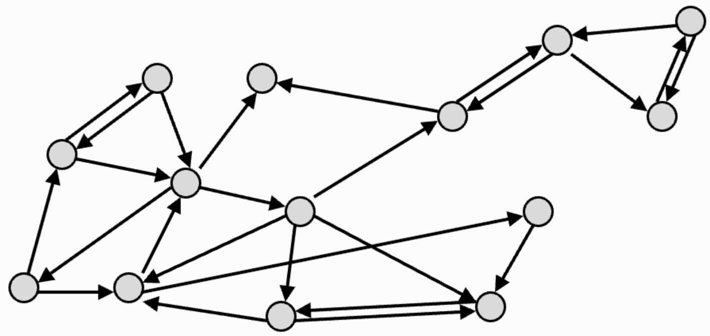
Directed Graphs

$G = (V, E)$

V is a set of vertices (an underlying set of elements)

E is a set of edges (ordered pairs of vertices; i.e. connections from one to the next).

Path $v_0, v_1, \dots, v_k$ such that $(v_i, v_{i+1}) \in E$
Simple Path: path with all v_i distinct
Cycle: path with $v_0 = v_k$ (and $k > 0$)
Simple Cycle: simple path plus edge (v_k, v_0) with $k > 0$



Draw the graph!

Let $G = (V, E)$.

V = {1, 2, 3, 4}

E = {(1, 1), (1, 2), (2, 3), (3, 4), (4, 1)}

Is there a cycle?

Is there a simple cycle?