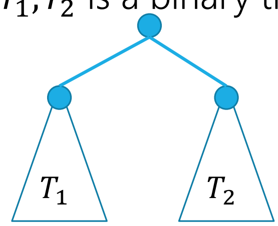
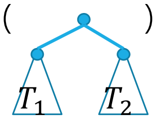
Binary Trees

Basis: A single node is a rooted binary tree.

Recursive Step: If T_1 and T_2 are rooted binary trees with roots r_1 and r_2, then a tree rooted at a new node, with children r_1, r_2 is a binary tree.

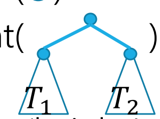


size()=1

size() =

size(T_1) + size(T_2) + 1

height() = 0

height() =

1+**max**(height(T_1),height(T_2))

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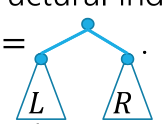
Structural Induction on Binary Trees (cont.)

Let $P(T)$ be "size(T) $\leq 2^{height(T)+1} - 1$". We show $P(T)$ for all binary trees T by structural induction.

$T =$  .

height(T)=$1 + \max\{height(L), height(R)\}$

size(T)= 1 +size(L)+size(R)

So $P(T)$ holds, and we have $P(T)$ for all binary trees T by the principle of induction.

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Structural Induction Template

1. Define $P()$ State that you will show $P(x)$ holds for all $x \in S$ and that your proof is by structural induction.

2. Base Case: Show $P(b)$
[Do that for every b in the basis step of defining S]

3. Inductive Hypothesis: Suppose $P(x)$
[Do that for every x listed as already in S in the recursive rules].

4. Inductive Step: Show $P()$ holds for the "new elements."
[You will need a separate step for every element created by the recursive rules].

5. Therefore $P(x)$ holds for all $x \in S$ by the principle of induction.

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Claim for all $x, y \in \Sigma^*$ len(x·y)=len(x) + len(y).

Let $P(y)$ be "len(x·y)=len(x) + len(y) for all $x \in \Sigma^*$."

We prove $P(y)$ for all $x \in \Sigma^*$ by structural induction.

Base Case:

Inductive Hypothesis

Inductive Step:

We conclude that $P(y)$ holds for all string y by the principle of induction. Unwrapping the definition of P, we get $\forall x \forall y \in \Sigma^*$ len(xy)=len(x)+len(y), as required.

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