Stamp Collection, Done Wrong

Define $P(n)$ I can make n cents of stamps with just 4 and 5 cent stamps.

We prove $P(n)$ is true for all $n \geq 12$ by induction on n.

Base Case:

12 cents can be made with three 4 cent stamps.
Inductive Hypothesis Suppose $P(k)$, $k \geq 12$.

Inductive Step:

We want to make $k + 1$ cents of stamps. By IH we can make k cents exactly with stamps. Replace one of the 4 cent stamps with a 5 cent stamp.

$P(n)$ holds for all n by the principle of induction.

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Making Induction Proofs Pretty

All of our induction proofs will come in 5 easy(?) steps!

1. Define $P(n)$. State that your proof is by induction on n.
2. Base Cases: Show $P(b_{min}), P(b_{min+1}) \dots P(b_{max})$ i.e. show the base cases
3. Inductive Hypothesis: Suppose $P(b_{min}) \land P(b_{min} + 1) \land \cdots \land P(k)$ for an arbitrary $k \geq b_{max}$. (The smallest value of k assumes **all** bases cases, but nothing else)
4. Inductive Step: Show $P(k + 1)$ (i.e. get $[\mathrm{P}(\mathrm{b}_{min}) \land \cdots \land P(k)] \to P(k + 1)$)
5. Conclude by saying $P(n)$ is true for all $n \geq b_{min}$ by the principle of induction.

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Fibonacci Inequality

Show that $f(n) \leq 2^n$ for all $n \geq 0$ by induction.

$f(0) = 1; \quad f(1) = 1$
$f(n) = f(n-1) + f(n-2)$ for all $n \in \mathbb{N}, n \geq 2$.

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Claim: $3|(2^{2n} - 1)$ for all $n \in \mathbb{N}$.

Let $P(n)$ be "$3|(2^{2n} - 1)$." We show $P(n)$ holds for all $n \in \mathbb{N}$.

Base Case ($n = 0$) note that $2^{2n} - 1 = 2^0 - 1 = 0$. Since $3 \cdot 0 = 0$, and 0 is an integer, $3|(2^{2 \cdot 0} - 1)$.

Inductive Hypothesis: Suppose $P(k)$ holds for an arbitrary $k \geq 0$

Inductive Step:

Target: $P(k+1)$, i.e. $3|(2^{2(k+1)} - 1)$

Therefore, we have $P(n)$ for all $n \in \mathbb{N}$ by the principle of induction.

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