Setup

$$g(n) = \begin{cases} 2 & \text{if } n = 2 \\ g(n-1)^2 + 3g(n-1) & \text{if } n > 2 \end{cases}$$

Let $P(n)$ be "$g(n)$ is even."

We show $P(n)$ for all integers $n \geq 2$ by induction on n.

Base Case ($\boldsymbol{n = 2}$): <todo>

Inductive Hypothesis: Suppose $P(k)$ holds for an arbitrary $\mathrm{k} \geq 2$.

Inductive Step:

<todo>

We conclude $P(k+1)$. Therefore, $P(n)$ holds for all $n \geq 2$ by the principle of induction.

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Making Induction Proofs Pretty

All of our induction proofs will come in 5 easy(?) steps!

1. Define $P(n)$. State that your proof is by induction on n.
2. Base Case: Show $P(b)$ i.e. show the base case
3. Inductive Hypothesis: Suppose $P(k)$ for an arbitrary $k \geq b$.
4. Inductive Step: Show $P(k+1)$ (i.e. get $P(k) \rightarrow P(k+1)$)
5. Conclude by saying $P(n)$ is true for all $n \geq b$ by the principle of induction.

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Induction on Primes.

Let $P(n)$ be "n can be written as a product of primes."

We show $P(n)$ for all integers $n \geq 2$ by induction on n.

Base Case ($\boldsymbol{n = 2}$): 2 is a product of just itself. Since 2 is prime, it is written as a product of primes.

Inductive Hypothesis: Suppose $P(k)$ holds for an arbitrary integer $k \geq 2$.

Inductive Step:
Case 1, $k + 1$ is prime: then $k + 1$ is automatically written as a product of primes.

Case 2, $k + 1$ is composite:

Therefore $P(k + 1)$.

$P(n)$ holds for all $n \geq 2$ by the principle of induction.

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Practical Advice

How many base cases do you need?

Always at least one.

If you're analyzing recursive code or a recursive function, at least one for each base case of the code/function.

If you always go back s steps, at least s consecutive base cases.

Enough to make sure every case is handled.

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