Proof By Contradiction

Claim: $\sqrt{2}$ is irrational (i.e. not rational).

Proof:

Rational

A real number x is rational if (and only if) there exist integers p and q, with $q \neq 0$ such that $x = p/q$.

`Rational` $(x) \coloneqq \exists p \exists q($ `Integer` $(p) \wedge$ `Integer` $(q) \wedge (x = p/q) \wedge q \neq 0)$

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What's the difference?

What's the difference between proof by contrapositive and proof by contradiction?

Show $p \to q$	**Proof by contradiction**	**Proof by contrapositive**
Starting Point	$\neg(p \to q) \equiv (p \wedge \neg q)$	$\neg q$
Target	Something false	$\neg p$

Show p	**Proof by contradiction**	**Proof by contrapositive**
Starting Point	$\neg p$	---
Target	Something false	---

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Another Proof By Contradiction

Claim: There are infinitely many primes.

Proof:

Suppose for the sake of contradiction, that there are only finitely many primes. Call them $p_1, p_2, \ldots, p_k$.

Consider the number $q = p_1 \cdot p_2 \cdot \cdots \cdot p_k + 1$

Case 1: q is prime

Case 2: q is composite

But [] is a contradiction! So there must be infinitely many primes.

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Just the Skeleton

"For all integers x, if x^2 is even, then x is even."

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