Inference Rules

Eliminate ∧

$$\frac{A \wedge B}{\therefore \quad A, B}$$

Eliminate ∨

$$\frac{A \vee B, \neg A}{\therefore \quad B}$$

Intro ∧

$$\frac{A, B}{\therefore \quad A \wedge B}$$

Intro ∨

$$\frac{A}{\therefore A \vee B, B \vee A}$$

Direct Proof rule

$$\frac{A \Rightarrow B}{A \rightarrow B}$$

Modus Ponens

$$\frac{P \rightarrow Q, P}{\therefore \quad Q}$$

You can still use all the propositional logic equivalences too!

Try it!

Given: $p \vee q$, $(r \wedge s) \rightarrow \neg q$, r.
Show: $s \rightarrow p$

Eliminate ∧

$$\frac{A \wedge B}{\therefore \quad A, B}$$

Eliminate ∨

$$\frac{A \vee B, \neg A}{\therefore \quad B}$$

Intro ∧

$$\frac{A; B}{\therefore \quad A \wedge B}$$

Intro ∨

$$\frac{A}{\therefore A \vee B, B \vee A}$$

Direct Proof rule

$$\frac{A \Rightarrow B}{A \rightarrow B}$$

Modus Ponens

$$\frac{P \rightarrow Q; P}{\therefore \quad Q}$$

You can still use all the propositional logic equivalences too!

Proof Using Quantifiers

Suppose we know $\exists x P(x)$ and $\forall y[\, P(y) \rightarrow Q(y)]$. Conclude $\exists x Q(x)$.

Eliminate $\forall$
$$\frac{\forall x\ P(x)}{\therefore P(a) \text{ for any } a}$$

Intro $\exists$
$$\frac{P(c) \text{ for some } c}{\therefore \quad \exists x\ P(x)}$$

Intro $\forall$
$$\frac{P(a);\, a \text{ is arbitrary}}{\therefore \qquad \forall x\ P(x)}$$

Eliminate $\exists$
$$\frac{\exists x P(x)}{\therefore \quad P(c) \text{ for a fresh } c}$$

Divides

Divides
For integers x, y we say $x \vert y$ ("x divides y") iff there is an integer z such that $xz = y$.

Which of these are true?

$2|4$ $\qquad$ $4|2$ $\qquad$ $2|-2$

$5|0$ $\qquad$ $0|5$ $\qquad$ $1|5$