Nested Quantifiers

$\forall x \exists y\ P(x, y)$

"For every x there exists a y such that $P(x, y)$ is true."

y might change depending on the x (people have different friends!).

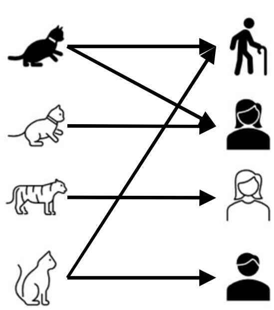
$\exists x \forall y\ P(x, y)$

"There is an x such that for all y, $P(x, y)$ is true."

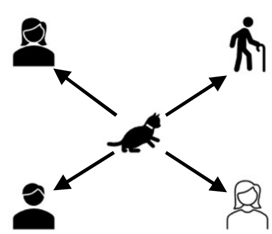
There's a special, magical x value so that $P(x, y)$ is true regardless of y.

Try it yourselves

Every cat loves some human.



There is a cat that loves every human.



Let your domain of discourse be mammals.
Use the predicates `Cat`(x), `Dog`(x), and `Loves`(x, y) to mean x loves y.

Our First Direct Proof

Definitions
$\text{Even}(x) \coloneqq \exists k(x = 2k)$

Prove: "For all integers x, if x is even, then x^2 is even." $\forall x\big(\text{Even}(x) \rightarrow \text{Even}(x^2)\big)$

Integer

We need a basic starting point to be able to prove things.

Objects to work with.

An integer: is any real number with no fractional part.

Some definitions to analyze

Even	Odd
`Even(x)` := An integer, x, is even if and only if there is an integer k such that $x = 2k$.	**`Odd(x)` := An integer, x, is odd if and only if there is an integer k such that $x = 2k + 1$.**