

## Back to the Compound Proposition...

"Robbie knows the Pythagorean Theorem if he is a mathematician and took geometry, and he is a mathematician or did not take geometry."

$$(p \text{ if } (q \wedge r)) \wedge (q \vee (\neg r))$$

$p$  "Robbie knows the Pythagorean Theorem"  
 $q$  "Robbie is a mathematician"  
 $r$  "Robbie took geometry"

What promise am I making?

$$((q \wedge r) \rightarrow p) \wedge (q \vee (\neg r))$$

$$(p \rightarrow (q \wedge r)) \wedge (q \vee (\neg r))$$

The first one! Being a mathematician and taking geometry is the condition. Knowing the Pythagorean Theorem is the promise.

## Law of Implication

Implications are hard.

AND/OR/NOT make more intuitive sense to me...

can we rewrite implications using just ANDs ORs and NOTs?

$p$	$q$	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

One approach: think "when is this implication false?" then negate it (you might want one of DeMorgan's Laws!

# Properties of Logical Connectives

You don't have to memorize this list!

These identities hold for all propositions  $p, q, r$

- **Identity**
  - $p \wedge T \equiv p$
  - $p \vee F \equiv p$
- **Domination**
  - $p \vee T \equiv T$
  - $p \wedge F \equiv F$
- **Idempotent**
  - $p \vee p \equiv p$
  - $p \wedge p \equiv p$
- **Commutative**
  - $p \vee q \equiv q \vee p$
  - $p \wedge q \equiv q \wedge p$
- **Associative**
  - $(p \vee q) \vee r \equiv p \vee (q \vee r)$
  - $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
- **Distributive**
  - $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
  - $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
- **Absorption**
  - $p \vee (p \wedge q) \equiv p$
  - $p \wedge (p \vee q) \equiv p$
- **Negation**
  - $p \vee \neg p \equiv T$
  - $p \wedge \neg p \equiv F$

## Our First Proof

$$(a \wedge b) \vee (\neg a \wedge b) \vee (\neg a \wedge \neg b) \equiv$$

None of the rules look like this

Practice of Proof-Writing:

**Big Picture**...WHY do we think this might be true?

The last two "pieces" came from the  $\equiv (\neg a \vee b)$  vacuous proof lines...maybe the " $\neg a$ " came from there? Maybe that **simplifies** down to  $\neg a$