

CSE 311 : Midterm Exam

1. Translation

Let your domain of discourse be positive integers.

For this problem, you may use the predicates

- $\text{Even}(x)$ which is true if and only if x is even.
- $\text{Odd}(x)$ which is true if and only if x is odd.
- $\text{PrimePower}(x)$ which is true if and only if x is “a prime power” (which means the prime factorization of x is p^a for a prime number p and an integer a . $125 = 5^3$ is a prime power, $24 = 2^3 \cdot 3$ is not a prime power)
- $\text{PowerOfTwo}(x)$, which is true if and only if x is a power of 2 (i.e., $x = 2^c$ for some integer c)
- standard math predicates (e.g. $=, \neq, <, >, \geq, \dots$)

(a) Translate the following predicate logic statement into English. Your translation must be natural.

$$\forall x(\text{PrimePower}(x) \rightarrow \text{Odd}(x) \vee \text{PowerOfTwo}(x))$$

(b) Translate “There is more than one prime power” into predicate logic.

(c) Find an equivalent statement to the one below by taking the contrapositive of the implication inside. Give your final answer in English; you do not need to show work.

“For every integer, if it is even then it is a power of two or not a prime power.”

(d) Negate the following predicate logic sentence. In your final answer, negations should only be applied to single predicates.

$$\exists x \forall y ([\text{Even}(y) \vee \text{PrimePower}(y)] \rightarrow [\text{Even}(x) \wedge \text{Odd}(y)])$$

2. Set or Number Theory Proof

For the questions below you may use the definitions of modular equivalence and divides and algebra.

You may not use results from the number theory formula sheet or theorems proven in class (though you may emulate those proofs!)

Finally, you may also use this fact without proving it:

Fact 1: For any two integers x, y and any prime p : if $xy \equiv 0 \pmod{p}$ then $p|x$ or $p|y$.

(a) Disprove this statement with a counterexample.

For every integer n , $ab \equiv 0 \pmod{n}$ implies $a \equiv 0 \pmod{n}$ or $b \equiv 0 \pmod{n}$. (Hint: you will need to choose n to be a composite number).

(b) Prove that for every prime p : If $ab \equiv 0 \pmod{p}$, then $a \equiv 0 \pmod{p}$ or $b \equiv 0 \pmod{p}$.

3. Induction Proof

Prove that $6 | (10^{2n} + 2)$ for all $n \in \mathbb{Z}^+$ using induction on n .

Recall that \mathbb{Z}^+ is the positive integers (i.e., starting at 1).

Don't forget to define your predicate as part of your proof!

Hint: $198 = 6 \cdot 33$.