Homework 4: English Proofs

Version 2: We corrected some typos in the line numbers in problem 5 (those were not the bug(s) you were looking for).
Due date: Wednesday, October 23rd at 11:59 PM
If you work with others (and you should!), remember to follow the collaboration policy outlined in the syllabus.
In general, you are graded on both the clarity and accuracy of your work. Your solution should be clear enough that someone in the class who had not seen the problem before would understand it.
We sometimes describe approximately how long our explanations are. These are intended to help you understand approximately how much detail we are expecting. You are allowed to have longer explanations, but explanations significantly longer than necessary may receive deductions.

Be sure to read the grading guidelines on the assignments page for more information on what we're looking for.

1. Being Direct [12 points]

Let the domain of discourse for this problem be integers. Define the predicates $\text{Odd}(x) := \exists k(x = 2k + 1)$, and $\text{Even}(x) := \exists k(x = 2k)$.

(a) Translate the following claim to predicate logic: [4 points]

For all odd integers n and m, $5n - 3m$ is even.

(b) Prove that the claim is true. [8 points] For this problem write an inference proof (the kind we did in Lectures 7 and 8). You can use the justification "Algebra" to do algebraic operation(s) and the justification "definition of even" or "definition of odd" to apply those definitions.

2. Oddly Even [12 points]

In this problem we will analyze the statement:

For all integers n, if $n - 5$ is odd, then n^2 must be even.

(a) Translate the claim into predicate logic. Let your domain of discourse be integers, and use the definitions in the previous problem. [4 points]

(b) Prove the claim is true by writing an English proof. [8 points]

3. The Oddacity [10 points]

When direct proofs fail, our logical equivalences can come to the rescue. Consider the statement

For every integer k, if $k^2 + 3$ is even then k is odd

Proving this directly is not easy (try it for yourself to see!). Instead, we will prove the contrapositive of this statement.

(a) Write the contrapositive of the given statement (in English). [2 points]

(b) Write a proof by contrapositive (do an English proof) of the given statement. [8 points]

4. Something is wrong here... [12 points]

4.1. Dance or Die! [6 points]

You have been kidnapped by pirates and are offered a choice: either beat the captain in a dance-off or walk the plank. (Specifically, if you beat the captain in a dance-off, you will not walk the plank.)

Assume the following things to be true.

- The captain gets to choose the style of dance used for the dance-off. Both participants will perform the same style.
- You are classically trained in ballet, tap, and contemporary dance, and are confident that you can beat the captain in these dance styles regardless of her skill. However, you will certainly lose in any other dance style.
- The captain tells you that she knows ballet and tap, and she would not lie.
- The captain will only pick a style of dance that she knows.

Consider the following (incorrect) proof of the claim: ”You will not walk the plank.” .

1. The captain gets to choose the style of dance, and will only pick a style of dance that she knows.
2. The captain only knows ballet and tap, so the dance-off must be ballet or tap.
3. You can beat the captain in both ballet and tap, so you will win in either scenario.
4. We conclude that you will win the dance-off.
5. Because you win the dance-off, you will not walk the plank.

(a) Identify the most significant error in the proof and discuss why this step is incorrect. Sentences have been labeled to easily refer back to specific portions of the proof.

4.2. What's wrong with this proof? [6 points]

Consider the following statement:

For all real numbers x, y, if $x^2 = y^2$, then $x = y$.

And the following spoof (incorrect proof) of the statement:

> Let x, y be arbitrary real numbers and suppose that $x^2 = y^2$. Taking the square root, we obtain $x = y$. Thus, our claim holds.

(a) Why is the above proof incorrect?

Here, let's try again. This must be correct this time, right?

> Let x and y be arbitrary real numbers and suppose that $x = y$. Squaring both sides of the equation, we obtain $x^2 = y^2$.

(b) Again, why is the above proof incorrect?

(c) Is the original statement true or false? If the statement is true, write a correct proof. If it is false, provide a counterexample.

5. More wrong proofs? [12 points]

Identify and explain all significant logical errors in the proofs below, listing the line number(s) where the error is first introduced. If the original claim is true, explain how to correct the proof; otherwise, provide a counterexample showing that the claim is false with brief justification (2-3 sentences).

By "significant" we mean an error which misapplies rules of inference. Referring to a wrong line number or unusual formatting, for example, are not significant.

To show a claim is false, one should provide a case in which all the givens are true, the conclusion evaluates to false. This may involve assigning truth values to propositions and defining predicates (so that quantified predicate logic statements have a well-defined truth value).

(a) This proof claims to show that $(p \to r) \land (q \to r)$ follows from $(p \land q) \to r$. There are 2 errors.

1.	$(p \land q) \to r$	Given
2.1.	p	Assumption
2.2.	q	Elim $\land$: 1
2.3.	$p \land q$	Intro $\land$: 2.1, 2.2
2.4.	r	Modus Ponens: 2.3, 1
2.	$p \to r$	Direct Proof
3.1.	q	Assumption
3.2.	$p \land q$	Intro $\land$: 2.1, 3.1
3.3.	r	Modus Ponens: 3.2, 1
3.	$q \to r$	Direct Proof
4.	$(p \to r) \land (q \to r)$	Intro $\land$: 2, 3

(b) Fix the domain of discourse to be all integers and let `P(x, y)` and `Q(x, y)` be predicates. The following proof claims that $\forall x\, \forall y$ `Q(x, y)` follows from $\forall x\, \exists y$ `P(x, y)` and $\forall x\, \forall y$ (`P(x, y)` $\to$ `Q(x, y)`). There is 1 error.

1.	$\forall x\, \exists y$ `P(x, y)`	Given
2.	$\forall x\, \forall y$ (`P(x, y)` $\to$ `Q(x, y)`)	Given
3.	Let x be arbitrary.	
4.	$\exists y$ `P(x, y)`	Elim $\forall$: 1
5.	`P(x, y)`	Elim $\exists$: 4
6.	$\forall y$ (`P(x, y)` $\to$ `Q(x, y)`)	Elim $\forall$: 2
7.	`P(x, y)` $\to$ `Q(x, y)`	Elim $\forall$: 6
8.	`Q(x, y)`	Modus Ponens: 5, 7
9.	$\forall y$ `Q(x, y)`	Intro $\forall$: 8
10.	$\forall x\, \forall y$ `Q(x, y)`	Intro $\forall$: 9

6. Divides [8 points]

Write an English proof to show that if 8 divides $(x + 5)$ (i.e. $8 | (x + 5)$) for an integer x, then x is odd. Recall that English proofs don't have domains of discourse, so you need to state the types for your variables when you introduce them.

7. In the Real World: Logic Beyond CS [6 points]

Background

Our main goal in 311 is to prepare you to be a better computer scientist, but some of the lessons of the course are useful in "everyday life." One of the most common errors in reasoning in the "real world" is confusing an implication for its converse. That is, thinking that $p \to q$ and $q \to p$ can always be interchanged. This error can appear in a multiple ways.

One common way is a mistake called "affirming the consequent." The error is from givens $p \to q$ and q to conclude p. That conclusion is an error – from q and $q \to p$ (the converse of $p \to q$), one can conclude p. But not from q and $p \to q$.

This mistake can also appear in much longer strings of logical reasoning. For example, it is tempting to combine "If it is raining, then we won't play softball."; "If we don't play softball, then we'll get ice cream"; "we got ice cream" into the conclusion that "it is raining". But might not be raining! (It could be that we get ice cream if we don't play softball OR if we play softball and win).

A common reason for this error is assuming that one cause is the only possible cause. Of course, this doesn't mean the conclusion is necessarily wrong—it may be that p is in fact true! But we can't guarantee it from just the reasoning given.

GRADING NOTE: These are open-ended question with the goal of seeing that 311 and logic can be seen in real life. Thus, this will be graded more leniently. We want you to focus more on finding examples of 311 in real life and having fun with this problem, rather than being stressed about if your answer is sufficient.

7.1. In Real Life [6 points]

Find an example of someone making an argument in real-life, and determine if they made an error involving mixing up an implication and its converse. This could be in the news (like a politician or opinion piece), in culture (like a TV clip or an advertisement), or just something your friend said. If you can't think of one, you may use one of the examples in the section below.

(a) Write down a quote of the person making the argument.

(b) If you can link us to a source, do so here (don't worry about formatting). If you can't (because there's no record of the statement), just say that.

(c) From the quote you have in (a), respond to the following:
(i) Define propositions that they are dealing with and translate what they say to propositional logic.
(ii) What givens are they asserting and what is their conclusion (i.e. what do they intend to argue for)?
(iii) Are they making an error? If so explain why. If not, what inference rule(s) could you apply to formalize their argument?
Since people tend to speak informally (rather than in precise logical arguments), you may have to read between the lines a bit when doing this part.

(d) Suggest replacing one (or more) of the implications with their converse in the propositional logic you have in (c). With the change, can one reach their desired conclusion without making any logical error? (1-3 sentences)

(e) Do you think the converse(s) you inserted in the previous part are true? Or at least "often true"?[1] Explain in 1-3 sentences.

[1]For example, it might not be true that "If I have my umbrella, then it is raining" (since I also bring my umbrella when it snows), but snow is so rare that the implication would 'often' be true.

Some options

You're encouraged to keep your eyes out for this error in real life! Or to think about places you might have seen it. If you cannot find one, you might choose one of these options instead:

- This cartoon about logical penguins
- This clip from The Simpsons.
- This clip from Sesame Street.
- The clip from the childrens' show "If you give a mouse a cookie" available on Ed.

8. Feedback [1 point]

Answer these questions on the separate gradescope box for this question.

Please keep track of how much time you spend on this homework and answer the following questions. This can help us calibrate future assignments and future iterations of the course, and can help you identify which areas are most challenging for you.

- How many hours did you spend working on this assignment (excluding any extra credit questions, if applicable)? Report your estimate to the nearest hour.
- Which problem did you spend the most time on?
- Any other feedback for us?

9. The ONE RING [Extra Credit]

You will submit this question to the separate gradescope box for "homework 4 extra credit."

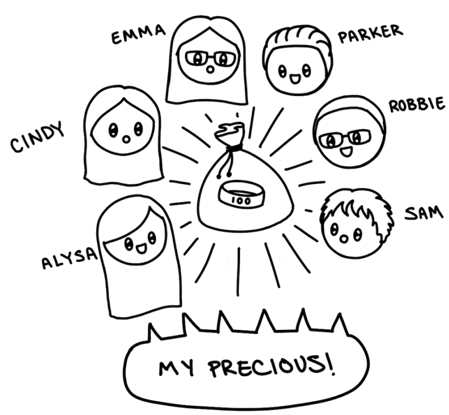

Figure 1: drawing of all hobbits, credit: Sierra Yee (honorable CSE 311 graduate)

Six Hobbits found a stash of 100 remnants of the ONE RING after it had been allegedly destroyed by Frodo. The Hobbits are coincidentally named Alysa, Cindy, Emma, Parker, Robbie, and Sam. They agreed to split the ring pieces using the following rules:

- The first Hobbit in alphabetical order becomes the leader of the Hobbits.
- The leader of Hobbits proposes how to split the remnants. For example, they might say "Alysa gets all 100 pieces of the ring, all other Hobbits get none" or "Alysa, Cindy, Emma, Parker, Robbie each gets 20 pieces, and Sam gets none."
- All Hobbits (including the leader) vote for or against the proposal.

- If 2 or more Hobbits disagree to the proposal, the Ring will possess the leader for being too greedy. Once possessed, the leader no longer participates in the splitting process (they do not vote, and they cannot receive ring remnants in the split).
- Otherwise, the ring remnants will be split as proposed.

Thus, the first round Alysa is the leader: if her proposal has been rejected by at least 2 Hobbits, she'd be possessed and Cindy becomes the leader, etc; If Alysa, Cindy, Emma, Parker, and Robbie get possessed, then Sam will become the leader and keep all the ring remnants.

The Hobbits' first priority is not to be possessed, since being possessed means they will forever be away from power. If they don't get possessed by the ring, they will try to get as many pieces of ring as possible for themselves, since the used-to-be nice Hobbits are corrupted at the sight of the omnipotent ring. Finally, in a scary world like this, every Hobbit tries to overpower others, so if they can get the same amount of ring remnants for agreeing and disagreeing with the proposal, they will disagree with the proposal and cause the leader to be possessed.

Assuming that all 6 Hobbits are smart (and greedy and all aware of the others' intelligence and greediness), what will happen?

Your solution should indicate which Hobbits will be possessed by the ring, and how many ring remnants each of the remaining Hobbits will receive.