CSE 311 Section 3

Propositions and Proofs

Administrivia & Introductions

Announcements & Reminders

- HW1
  - If you think something was graded incorrectly, submit a regrade request!

- HW2 due yesterday 10PM on Gradescope
  - Use late days if you need them!

- HW3
  - Due Wednesday 1/25 @ 10pm

References

- Helpful reference sheets can be found on the course website!
  - https://courses.cs.washington.edu/courses/cse311/23wi/resources/
- How to LaTeX (found on Assignments page of website):
  - https://courses.cs.washington.edu/courses/cse311/23wi/assignments/HowToLaTeX.pdf
- Equivalence Reference Sheet
  - https://courses.cs.washington.edu/courses/cse311/23wi/resources/reference-logical_equiv.pdf
  - https://courses.cs.washington.edu/courses/cse311/23wi/resources/logicalConnectPoster.pdf
- Boolean Algebra Reference Sheet
  - https://courses.cs.washington.edu/courses/cse311/23wi/resources/reference-boolean-alg.pdf
- Plus more!

Predicates & Quantifiers

Predicates & Quantifiers Review

- **Predicate**: a function that outputs true or false
  - Cat(x) := "x is a cat"
  - LessThan(x, y) := "$x < y$"

- **Domain of Discourse**: the types of inputs allowed in predicates
  - Numbers, mammals, cats and dogs, people in this class, etc.

- **Quantifiers**
  - Universal Quantifier $\forall x$: for all x, for every x
  - Existential Quantifier $\exists x$: there is an x, there exists an x, for some x

- **Domain Restriction**
  - Universal Quantifier ∀x: add the restriction as the hypothesis to an **implication**
  - Existential Quantifier $\exists x$: **AND** in the restriction

Problem 1 – Domain Restriction

Translate each of the following sentences into logical notation. These translations require some of our quantifier tricks. You may use the operators + and · which take two numbers as input and evaluate to their sum or product, respectively.

a) Domain: Positive integers; Predicates: *Even, Prime, Equal*
"There is only one positive integer that is prime and even."

b) Domain: Real numbers; Predicates: *Even, Prime, Equal*
"There are two different prime numbers that sum to an even number."

c) Domain: Real numbers; Predicates: *Even, Prime, Equal*
"The product of two distinct prime numbers is not prime."

d) Domain: Real numbers; Predicates: *Even, Prime, Equal, Positive, Greater, Integer*
"For every positive integer, there is a greater even integer"

Work on parts (a) and (b) with the people around you, and then we'll go over it together!

Problem 1 – Domain Restriction

a) Domain: Positive integers; Predicates: *Even, Prime, Equal*
“There is only one positive integer that is prime and even.”

Problem 1 – Domain Restriction

b) Domain: Real numbers; Predicates: *Even, Prime, Equal*

“There are two different prime numbers that sum to an even number.”

Problem 2 – ctrl-z

Translate these logical expressions to English. For each of the translations, assume that domain restriction is being used and take that into account in your English versions.

Let your domain be all UW Students. Predicates $\text{143Student}(x)$ and $\text{311Student}(x)$ mean the student is in CSE 143 and 311, respectively. $\text{BioMajor}(x)$ means x is a bio major, $\text{DidHomeworkOne}(x)$ means the student did homework 1 (of 311). Finally $\text{KnowsJava}(x)$ and $\text{KnowsDeMorgan}(x)$ mean x knows Java and knows DeMorgan's Laws, respectively.

a) $\forall x\big(\text{143Student}(x) \rightarrow \text{KnowsJava}(x)\big)$

b) $\exists x\big(\text{143Student}(x) \wedge \text{BioMajor}(x)\big)$

c) $\forall x([\text{311Student}(x) \wedge \text{DidHomeworkOne}(x)] \rightarrow \text{KnowsDeMorgan}(x))$

Work on parts (a) and (c) with the people around you, and then we'll go over it together!

Problem 2 – ctrl-z

a) $\forall x(\text{143Student}(x) \rightarrow \text{KnowsJava}(x))$

Problem 2 – ctrl-z

c) $\forall x([\text{311Student}(x) \land \text{DidHomeworkOne}(x)] \rightarrow \text{KnowsDeMorgan}(x))$

Problem 4 – Quantifier Switch

Consider the following pairs of sentences. For each pair, determine if one implies the other, if they are equivalent, or neither.

a) $\forall x\, \forall y\, P(x, y)$ $\quad$ $\forall y\, \forall x\, P(x, y)$

b) $\exists x\, \exists y\, P(x, y)$ $\quad$ $\exists y\, \exists x\, P(x, y)$

c) $\forall x\, \exists y\, P(x, y)$ $\quad$ $\forall y\, \exists x\, P(x, y)$

d) $\forall x\, \exists y\, P(x, y)$ $\quad$ $\exists x\, \forall y\, P(x, y)$

e) $\forall x\, \exists y\, P(x, y)$ $\quad$ $\exists y\, \forall x\, P(x, y)$

Work on parts (d) and (e) with the people around you, and then we'll go over it together!

Problem 4 – Quantifier Switch

d) $\forall x \, \exists y \, P(x, y)$ $\exists x \, \forall y \, P(x, y)$

Problem 4 – Quantifier Switch

e) $\forall x\ \exists y\ P(x, y)$ $\exists y\ \forall x\ P(x, y)$

Formal Proofs

Inference Proofs

- New way of doing proofs:
  - Write down all the facts we know (givens)
  - Combine the things we know to derive new facts
  - Continue until what we want to show is a fact

- **Modus Ponens**
  - $[(p \rightarrow q) \land p] \rightarrow q \equiv T$
  - If you have an implication and its hypothesis as facts, you can get the conclusion

- **Direct Proof Rule**
  - Assume x and then eventually get y, you can conclude that $x \rightarrow y$

Inference Proof Example

Given $((p \to q) \wedge (q \to r))$, show that $(p \to r)$

1.	$((p \to q) \wedge (q \to r))$		Given
2.	$p \to q$		Eliminate $\wedge$: 1
3.	$q \to r$		Eliminate $\wedge$: 1
	4.1	p	Assumption
	4.2	q	Modus Ponens: 4.1, 2
	4.3	r	Modus Ponens: 4.2, 3
5.	$p \to r$		Direct Proof Rule

Problem 5 – Formal Proof (Direct Proof Rule)

Show that $\neg t \rightarrow s$ follows from $t \vee q$, $q \rightarrow r$ and $r \rightarrow s$

Work on this problem with the people around you, and then we'll go over it together!

Problem 6 – Find The Bug

Each of these inference proofs is incorrect. Identify the line (or lines) which incorrectly apply a law, and explain the error. Then, if the claim is false, give concrete examples of propositions to show it is false. If it is true, write a correct proof.

a) This proof claims to show that given $a \to (b \lor c)$, we can conclude $a \to c$.

1.	$a \to (b \lor c)$	Given
2.1	a	Assumption
2.2	$\neg b$	Assumption
2.3	$b \lor c$	Modus Ponens: 1, 2.1
2.4	c	Eliminate $\lor$: 2.2, 2.3
3.	$a \to c$	Direct Proof Rule

Problem 6 – Find The Bug

Each of these inference proofs is incorrect. Identify the line (or lines) which incorrectly apply a law, and explain the error. Then, if the claim is false, give concrete examples of propositions to show it is false. If it is true, write a correct proof.

b) This proof claims to show that given $p \to q$ and r, we can conclude $p \to (q \vee r)$.

1.	$p \to q$	Given
2.	r	Given
3.	$p \to (q \vee r)$	Intro $\vee$: 1, 2

Problem 6 – Find The Bug

Each of these inference proofs is incorrect. Identify the line (or lines) which incorrectly apply a law, and explain the error. Then, if the claim is false, give concrete examples of propositions to show it is false. If it is true, write a correct proof.

c) This proof claims to show that given $p \rightarrow q$ and q that we can conclude p.

1.	$p \rightarrow q$	Given
2.	q	Given
3.	$\neg p \lor q$	Law of Implication: 1
4.	p	Eliminate $\lor$: 2, 3

That's All, Folks!

Thanks for coming to section this week!
Any questions?