

Bijection

One-to-one (aka injection)

A function f is one-to-one iff
 $\forall a \forall b (f(a) = f(b) \rightarrow a = b)$

Onto (aka surjection)

A function $f: A \rightarrow B$ is onto iff
 $\forall b \in B \exists a \in A (b = f(a))$

Bijection

A function $f: A \rightarrow B$ is a bijection iff
 f is one-to-one and onto

A bijection maps every element of the domain to **exactly** one element of the co-domain, and every element of the domain to **exactly** one element of the domain.

Some infinite sets

Two sets A, B have the same size (same cardinality)
 if and only if there is a bijection $f: A \rightarrow B$

Let's compare the sizes of: \mathbb{N} , \mathbb{Z} , $\{x : x \text{ is an even integer}\}$

[Pollev.com/robbie](https://pollev.com/robbie)

Proof that $[0,1)$ is not countable

Suppose, for the sake of contradiction, that there is a list of them:

Number	Digits after decimal	0	1	2	3	4	5	6	7	...
$f(0)$	0.	3	3	3	3	3	3	3	3	...
$f(1)$	0.	2	7	2	7	2	8	5	4	...
$f(2)$	0.	1	4	1	5	9	2	6	5	...
$f(3)$	0.	2	2	2	2	2	2	2	2	...
$f(4)$	0.	1	2	3	4	5	6	7	8	...
$f(5)$	0.	9	8	7	6	5	4	3	2	...
$f(6)$	0.	8	2	7	6	4	5	7	4	...
$f(7)$	0.	5	9	4	2	7	5	1	7	...
...

Our Second big takeaway

There are more functions $g: \mathbb{N} \rightarrow B$ than there are Java programs to compute them.

Some function must be **uncomputable**.

That is there is no piece of code which tells you the output of the function when you give it the appropriate input.