Try a few of your own

Decide whether each of these relations are

Reflexive, symmetric, antisymmetric, and transitive.

$\subseteq$ on $\mathcal{P}(\mathcal{U})$

$\geq$ on $\mathbb{Z}$

$>$ on $\mathbb{R}$

$|$ on $\mathbb{Z}^+$

$|$ on $\mathbb{Z}$

$\equiv (mod\ 3)$ on $\mathbb{Z}$

Symmetry: for all $\boldsymbol{a}, \boldsymbol{b} \in \boldsymbol{S}$, $[(\boldsymbol{a}, \boldsymbol{b}) \in \boldsymbol{R} \rightarrow (\mathrm{b}, \mathrm{a}) \in \boldsymbol{R}]$

Antisymmetry: for all $\boldsymbol{a}, \boldsymbol{b} \in \boldsymbol{S}$, $[(\boldsymbol{a}, \boldsymbol{b}) \in \boldsymbol{R} \wedge \boldsymbol{a} \neq \boldsymbol{b} \rightarrow (\mathrm{b}, \mathrm{a}) \notin \boldsymbol{R}]$

Transitivity: for all $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c} \in \boldsymbol{S}$, $[(\boldsymbol{a}, \boldsymbol{b}) \in \boldsymbol{R} \wedge (\boldsymbol{b}, \boldsymbol{c}) \in \boldsymbol{R} \rightarrow (\mathrm{a}, \mathrm{c}) \in \boldsymbol{R}]$

Reflexivity: for all $\boldsymbol{a} \in \boldsymbol{S}$, $[(\boldsymbol{a}, \boldsymbol{a}) \in \boldsymbol{R}]$

Two Prototype Relations

A lot of fundamental relations follow one of two prototypes:

Equivalence Relation

A relation that is reflexive, symmetric, and transitive is called an "equivalence relation"

Partial Order Relation

A relation that is reflexive, antisymmetric, and transitive is called a "partial order"

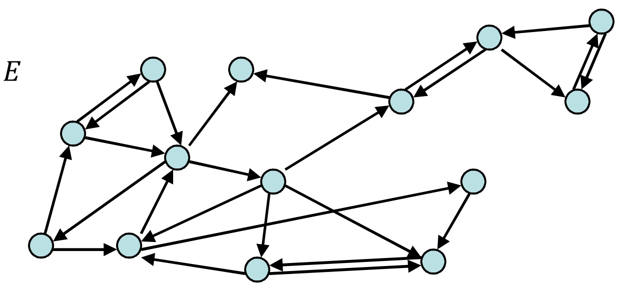
Directed Graphs

$G = (V, E)$

V is a set of vertices (an underlying set of elements)

E is a set of edges (ordered pairs of vertices; i.e. connections from one to the next).

Path $v_0, v_1, \dots, v_k$ such that $(v_i, v_{i+1}) \in E$
Simple Path: path with all v_i distinct
Cycle: path with $v_0 = v_k$ (and $k > 0$)
Simple Cycle: simple path plus edge (v_k, v_0) with $k > 0$



Combining Relations

If $\boldsymbol{S} = \{(\mathbf{2},\mathbf{2}), (\mathbf{2},\mathbf{3}), (\mathbf{3},\mathbf{1})\}$ and $\boldsymbol{R} = \{(\mathbf{1},\mathbf{2}), (\mathbf{2},\mathbf{1}), (\mathbf{1},\mathbf{3})\}$
Compute $\boldsymbol{S} \circ \boldsymbol{R}$ i.e. every pair (a, c) with a b with $(a, b) \in R$ and $(b, c) \in S$