Structural Induction Template

1. Define $P()$ Show that $P(x)$ holds for all $x \in S$. State your proof is by structural induction.
2. Base Case: Show $P(x)$ for all base cases x in S.
3. Inductive Hypothesis: Suppose $P(x)$ for all x listed as in S in the recursive rules.
4. Inductive Step: Show $P()$ holds for the “new element” given.

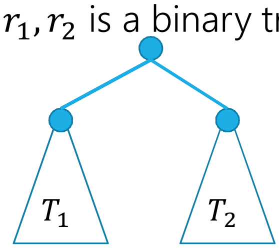
You will need a separate step for every rule.

5. Therefore $P(x)$ holds for all $x \in S$ by the principle of induction.

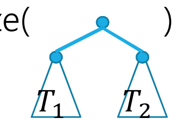
Binary Trees

Basis: A single node is a rooted binary tree.

Recursive Step: If T_1 and T_2 are rooted binary trees with roots r_1 and r_2, then a tree rooted at a new node, with children r_1, r_2 is a binary tree.

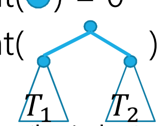


size(●)=1

size() =

size(T_1) + size(T_2) + 1

height(●) = 0

height() =

1+**max**(height(T_1),height(T_2))

Regular Expressions

Basis:

ε is a regular expression. The empty string itself matches the pattern (and nothing else does).

$\emptyset$ is a regular expression. No strings match this pattern.

a is a regular expression, for any $a \in \Sigma$ (i.e. any character). The character itself matching this pattern.

Recursive

If A, B are regular expressions then $(A \cup B)$ is a regular expression
matched by any string that matches A or that matches B [or both]).

If A, B are regular expressions then AB is a regular expression.
matched by any string x such that $x = yz$, y matches A and z matches B.

If A is a regular expression, then A^* is a regular expression.
matched by any string that can be divided into 0 or more strings that match A.

More Examples

$(0^*1^*)^*$

0^*1^*

$(0 \cup 1)^*(00 \cup 11)^*(0 \cup 1)^*$

$(00 \cup 11)^*$