Recursive Definitions of Sets

Q1: What is this set?

Basis Step: $6 \in S, 15 \in S$

Recursive Step: If $x, y \in S$ then $x + y \in S$

Q2: Write a recursive definition for the set of powers of 3 $\{1,3,9,27, \ldots\}$

Basis Step:

Recursive Step:

Structural Induction Template

1. Define $P()$ Show that $P(x)$ holds for all $x \in S$. State your proof is by structural induction.

2. Base Case: Show $P(x)$ for all base cases x in S.

3. Inductive Hypothesis: Suppose $P(x)$ for all x listed as in S in the recursive rules.

4. Inductive Step: Show $P()$ holds for the “new element” given.

You will need a separate step for every rule.

5. Therefore $P(x)$ holds for all $x \in S$ by the principle of induction.

Strings

Σ will be an **alphabet** the set of all the letters you can use in strings.

E.g. $\Sigma = \{0,1\}$

E.g. $\Sigma = \{\mathrm{a, b, c, \ldots, z, \_}\}$

Σ^* is the set of all **strings** you can build off of the letters.

E.g. if $\Sigma = \{0,1\}$, then $01001 \in \Sigma^*$

E.g. if $\Sigma = \{\mathrm{a, b, c, \ldots, z, \_}\}$, then $\mathrm{i\_love\_recursive\_sets} \in \Sigma^*$

ε is the **empty string**

Analogous to "" in Java

Functions on Strings

Length:

$\mathrm{len}(\varepsilon) = 0$

$\mathrm{len}(wa) = \mathrm{len}(w) + 1$ for $w \in \Sigma^*$, $a \in \Sigma$

Reversal:

$\varepsilon^R = \varepsilon$;
$(wa)^R = aw^R$ for $w \in \Sigma^*$, $a \in \Sigma$

Number of $\boldsymbol{c}$'s in a string

$\#_c(\varepsilon) = 0$
$\#_c(wc) = \#_c(w) + 1$ for $w \in \Sigma^*$;
$\#_c(wa) = \#_c(w)$ for $w \in \Sigma^*, a \in \Sigma \setminus \{c\}$.