Quantifiers

Writing implications can be tricky when we change the domain of discourse.

For every cat: if the cat is fat, then it is happy.

Domain of Discourse: cats $\forall x[\texttt{Fat}(x) \rightarrow \texttt{Happy}(x)]$

What if we change our domain of discourse to be all mammals?
We need to limit x to be a cat. How do we do that?

$\forall x[(\texttt{Cat}(x) \wedge \texttt{Fat}(x)) \rightarrow \texttt{Happy}(x)]$ $\forall x[\texttt{Cat}(x) \wedge (\texttt{Fat}(x) \rightarrow \texttt{Happy}(x))]$

Nested Quantifiers

Let our domain of discourse be $\{A, B, C, D, E\}$

And our proposition $P(x, y)$ be given by the table.

What should we look for in the table?

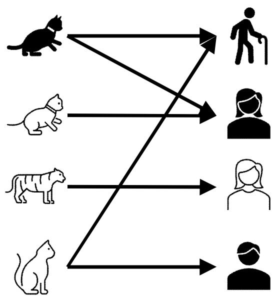
$\exists x \forall y P(x, y)$

$\forall x \exists y P(x, y)$

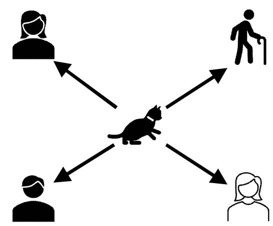
$P(x, y)$ (rows: x, columns: y)	A	B	C	D	E
A	T	T	T	T	T
B	T	F	F	T	F
C	F	T	F	F	F
D	F	F	F	F	T
E	F	F	F	T	F

Try it yourselves

Every cat loves some human.



There is a cat that loves every human.



Let your domain of discourse be mammals.
Use the predicates `Cat`(x), `Dog`(x), and `Loves`(x, y) to mean x loves y.

Try it!

Given: $p \vee q$, $(r \wedge s) \rightarrow \neg q$, r.
Show: $s \rightarrow p$

Eliminate ∧
$$\frac{A \wedge B}{\therefore\ A, B}$$

Eliminate ∨
$$\frac{A \vee B, \neg A}{\therefore\ B}$$

Intro ∧
$$\frac{A; B}{\therefore\ A \wedge B}$$

Intro ∨
$$\frac{A}{\therefore A \vee B, B \vee A}$$

Direct Proof rule
$$\frac{A \Rightarrow B}{A \rightarrow B}$$

Modus Ponens
$$\frac{P \rightarrow Q; P}{\therefore\ Q}$$

You can still use all the propositional logic equivalences too!