

# Warm Up

Translate this sentence into symbolic logic, and describe a weather pattern and transportation method that causes the proposition to be false.

It is snowing today, and if it is raining or snowing then we won't walk to school.

# Warm Up – Solution

Translate this sentence into symbolic logic, and describe a weather pattern and transportation method that causes the proposition to be false.

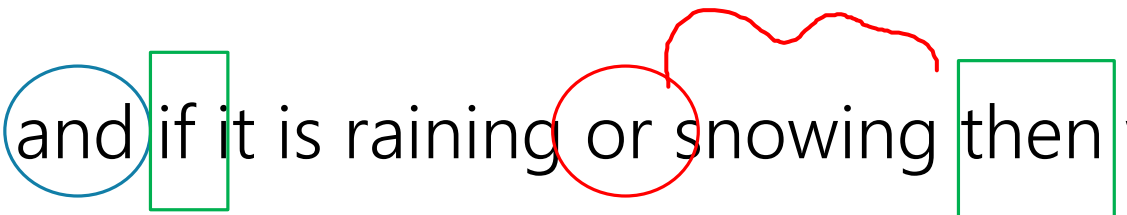
It is snowing today, and if it is raining or snowing then we won't walk to school.

Robbie's process: identify connecting words, identify propositions, figure out parentheses.

# Warm Up – Solution

Translate this sentence into symbolic logic, and describe a weather pattern and transportation method that causes the proposition to be false.

It is snowing today, and if it is raining or snowing then we won't walk to school.



Identify connecting words: look for and, or, not, if-then, etc.

# Warm Up – Solution

Translate this sentence into symbolic logic, and describe a weather pattern and transportation method that causes the proposition to be false.

$p$   $q$   $p$   $\neg r$   
It is snowing today, and if it is raining or snowing then we won't walk to school.

Identify propositions: What's left are propositions, look for repeats and hidden negations.

$p$ : it is snowing today.

$q$ : it is raining.

$r$ : we walk to school.

# Warm Up – Solution

Translate this sentence into symbolic logic, and describe a weather pattern and transportation method that causes the proposition to be false.

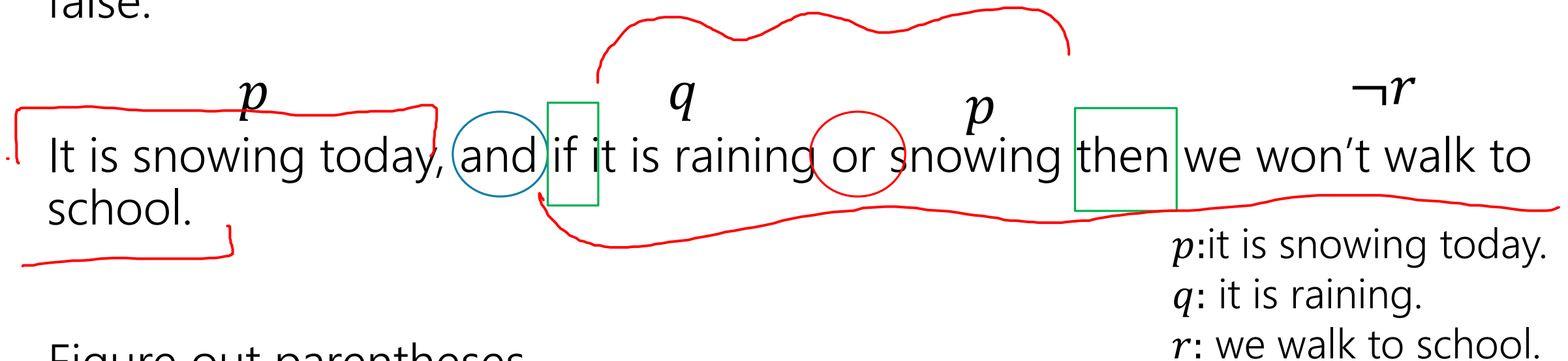


Figure out parentheses

$$p \wedge [(q \vee p) \rightarrow \neg r]$$

"raining or snowing" is the condition of the implication, not walking to school is the conclusion. Omitted words in other clauses hint that "It is snowing today" stands on its own.



[xkcd.com/1153](http://xkcd.com/1153)

# Our First Proof and Digital Logic

CSE 311 Winter 2023  
Lecture 3

# Announcements: Office Hours

Yael's at  
today will be  
12:30 - 2:30

A chance to talk to staff about the class.

Advice:

You don't have to have a question to come to office hours! (It can help, though)

Aim early on in the "homework cycle" (i.e., long before the deadline).

We won't answer every kind of question in office hours:

We don't "pregrade" homework. We won't tell you whether something is right or wrong.

We will help on homework, but not usually by "giving hints." We'll usually ask questions about what you've tried, ask questions to help you decide what to do next, or point you to good examples to look at.

You're allowed to talk to others at office hours, as long as you're still following the collaboration policy.

# Announcements: Section Materials

Handouts and solutions from sections are on the calendar.

Section solutions should be one of your most-used resources.

They're staff-written solutions for problems that are often similar to homeworks.

When you're wondering how much explanation to give, or what kind of formatting we might expect, section solutions are the first place to look (along with lecture slides).



# Homework Submissions

Make sure we can read what you submit.

We can't spend 5 minutes per submission deciding if that's a  $p$  or a  $q$ .

Typesetting guarantees we can read it.

Microsoft Word's equation editor is now halfway decent!

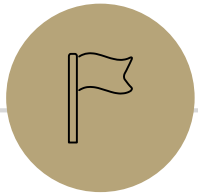
LaTeX is the industry standard for typesetting (if you go to CS grad school, you'll use it for all your papers). Overleaf is the easiest way to get started.

Need to know the code for a symbol? [Detexify](#)! Word uses LaTeX codes...mostly...

# Today

Our first proof!

Contrapositives and digital logic.



## Our First Proof

---

# Last Time

We showed

DeMorgan's Laws:

$$\neg(p \vee q) \equiv \neg p \wedge \neg q \text{ and } \neg(p \wedge q) \equiv \neg p \vee \neg q$$


# Law of Implication

Implications are hard.

AND/OR/NOT make more intuitive sense to me...

can we rewrite implications using just ANDs ORs and NOTs?

$p$	$q$	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

One approach: think "when is this implication false?"  
then negate it (you might want one of DeMorgan's Laws!)

$$\neg(p \wedge \neg q)$$
$$\neg p \vee \neg \neg q$$
$$\neg p \vee q$$

# Law of Implication

Implications are hard.

AND/OR/NOT make more intuitive sense to me...

can we rewrite implications using just ANDs ORs and NOTs?

$p$	$q$	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Seems like we might want  $\neg(p \wedge \neg q)$

$$\neg p \vee q$$

Seems like a reasonable guess.

So is it true? Is  $\neg p \vee q \equiv p \rightarrow q$ ?

# Law of Implication

$$\neg p \vee q \equiv p \rightarrow q$$

$p$	$q$	$p \rightarrow q$	$\neg p$	$\neg p \vee q$	$(\neg p \vee q) \leftrightarrow (p \rightarrow q)$
T	T	T	F	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

# Properties of Logical Connectives

We've derived two facts about logical connectives.

There's a lot more. A LOT more.

The next slide is a list of a bunch of them...

Most of these are much less complicated than the last two, so we won't go through them in detail.

DO NOT freak out about how many there are. We will always provide you the list on the next slide (no need to memorize).



# Properties of Logical Connectives

We will always give you this list!

For every propositions  $p, q, r$  the following hold:

- **Identity**

- $p \wedge T \equiv p$

- $p \vee F \equiv p$

- **Domination**

- $p \vee T \equiv T$

- $p \wedge F \equiv F$

- **Idempotent**

- $p \vee p \equiv p$

- $p \wedge p \equiv p$

- **Commutative**

- $p \vee q \equiv q \vee p$

- $p \wedge q \equiv q \wedge p$

- **Associative**

- $(p \vee q) \vee r \equiv p \vee (q \vee r)$

- $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

- **Distributive**

- $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

- $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

- **Absorption**

- $p \vee (p \wedge q) \equiv p$

- $p \wedge (p \vee q) \equiv p$

- **Negation**

- $p \vee \neg p \equiv T$

- $p \wedge \neg p \equiv F$

# Using Our Rules

WOW that was a lot of rules.

Why do we need them? Simplification!

Let's go back to the "law of implication" example.

$p$	$q$	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

When is the implication true? Just "or" each of the three "true" lines!

$$(p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q)$$

Also seems pretty reasonable

So is  $(p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q) \equiv (\neg p \vee q)$

i.e. are these both alternative representations of  $p \rightarrow q$ ?

# Our First Proof

We could make another truth table (you should! It's a good exercise)

But we have another technique that is nicer.

Let's try that one

Then talk about why it's another good option.

We're going to give an iron-clad guarantee that:

$$(p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q) \equiv \neg p \vee q$$

i.e. that this is another valid "law of implication"

# Our First Proof

How do we write a proof?

It's not always plug-and-chug...we'll be highlighting strategies throughout the quarter.

To start with:

Make sure we know what we want to show...

# Our First Proof

$$(p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q) \equiv$$

None of the rules look like this

Practice of Proof-Writing:

**Big Picture**...WHY do we think this might be true?

The last two "pieces" came from the vacuous proof lines...maybe the " $\neg p$ " came from there? Maybe that **simplifies** down to  $\neg p$

$$\equiv (\neg p \vee q)$$

# Let's apply a rule


$$(\neg p \wedge q) \vee (\neg p \wedge \neg q)$$

The law says:

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

$$\neg p \wedge (q \vee \neg q)$$
$$(\neg p \wedge q) \vee (\neg p \wedge \neg q) \equiv \neg p \wedge (q \vee \neg q)$$

# Our First Proof

$$(p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q) \equiv$$


None of the rules look like this

Practice of Proof-Writing:

**Big Picture**...WHY do we think this might be true?

The last two “pieces” came from the  $\equiv (\neg p \vee q)$   
vacuous proof lines...maybe the “ $\neg p$ ”  
came from there? Maybe that  
**simplifies** down to  $\neg p$

# Our First Proof

$$(p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q) \equiv (p \wedge q) \vee [(\neg p \wedge q) \vee (\neg p \wedge \neg q)]$$

Set ourselves an intermediate goal.  
Let's try to simplify those last two  
pieces

**Associative law**

Connect up the things we're working on.

$$\equiv (\neg p \vee q)$$



# Our First Proof

$$\begin{aligned}(p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q) &\equiv (p \wedge q) \vee [(\neg p \wedge q) \vee (\neg p \wedge \neg q)] \\ &\equiv (p \wedge q) \vee [\neg p \wedge (q \vee \neg q)]\end{aligned}$$

Set ourselves an intermediate goal.  
Let's try to simplify those last two  
pieces

**Distributive law**

We think  $\neg p$  is important, let's isolate it.

$$\equiv (\neg p \vee q)$$

# Our First Proof

$$\begin{aligned}(p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q) &\equiv (p \wedge q) \vee [(\neg p \wedge q) \vee (\neg p \wedge \neg q)] \\ &\equiv (p \wedge q) \vee [\neg p \wedge (q \vee \neg q)] \\ &\equiv (p \wedge q) \vee [\neg p \wedge \text{T}]\end{aligned}$$

Set ourselves an intermediate goal.  
Let's try to simplify those last two  
pieces

**Negation**

Should make things simpler.

$$\equiv (\neg p \vee q)$$

# Our First Proof

$$\begin{aligned}(p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q) &\equiv (p \wedge q) \vee [(\neg p \wedge q) \vee (\neg p \wedge \neg q)] \\ &\equiv (p \wedge q) \vee [\neg p \wedge (q \vee \neg q)] \\ &\equiv (p \wedge q) \vee [\neg p \wedge \text{T}] \\ &\equiv (p \wedge q) \vee [\neg p]\end{aligned}$$

Set ourselves an intermediate goal.  
Let's try to simplify those last two  
pieces

**Identity**

Should make things simpler.

$$\equiv (\neg p \vee q)$$

# Our First Proof

$$(p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q) \equiv (p \wedge q) \vee [(\neg p \wedge q) \vee (\neg p \wedge \neg q)]$$

$$\equiv (p \wedge q) \vee [\neg p \wedge (q \vee \neg q)]$$

$$\equiv (p \wedge q) \vee [\neg p \wedge \text{T}]$$

$$\equiv (p \wedge q) \vee [\neg p]$$

$$\equiv [\neg p] \vee (p \wedge q)$$

Stay on target:

We met our intermediate goal.

Don't forget the final goal!

We want to end up at  $(\neg p \vee q)$

If we apply the distribution rule,

We'd get a  $(\neg p \vee q)$

$$\equiv (\neg p \vee q)$$

# Our First Proof

$$\begin{aligned}(p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q) &\equiv (p \wedge q) \vee [(\neg p \wedge q) \vee (\neg p \wedge \neg q)] \\ &\equiv (p \wedge q) \vee [\neg p \wedge (q \vee \neg q)] \\ &\equiv (p \wedge q) \vee [\neg p \wedge \text{T}] \\ &\equiv (p \wedge q) \vee [\neg p] \\ &\equiv [\neg p] \vee (p \wedge q)\end{aligned}$$

Stay on target:

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We want to end up at  $(\neg p \vee q)$

If we apply the distribution rule,

We'd get a  $(\neg p \vee q)$

**Commutative**

$$\equiv (\neg p \vee q)$$

Make the expression look exactly like the law (more on this later)

# Our First Proof

$$\begin{aligned}(p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q) &\equiv (p \wedge q) \vee [(\neg p \wedge q) \vee (\neg p \wedge \neg q)] \\ &\equiv (p \wedge q) \vee [\neg p \wedge (q \vee \neg q)] \\ &\equiv (p \wedge q) \vee [\neg p \wedge \text{T}] \\ &\equiv (p \wedge q) \vee [\neg p] \\ &\equiv [\neg p] \vee (p \wedge q) \\ &\equiv (\neg p \vee p) \wedge (\neg p \vee q)\end{aligned}$$

Stay on target:

We met our intermediate goal.

Don't forget the final goal!

We want to end up at  $(\neg p \vee q)$

If we apply the distributive rule,

We'd get a  $(\neg p \vee q)$

Distributive

$$\equiv (\neg p \vee q)$$

Creates the  $(\neg p \vee q)$  we were hoping for.

# Our First Proof

$$\begin{aligned}(p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q) &\equiv (p \wedge q) \vee [(\neg p \wedge q) \vee (\neg p \wedge \neg q)] \\ &\equiv (p \wedge q) \vee [\neg p \wedge (q \vee \neg q)] \\ &\equiv (p \wedge q) \vee [\neg p \wedge \text{T}] \\ &\equiv (p \wedge q) \vee [\neg p] \\ &\equiv [\neg p] \vee (p \wedge q) \\ &\equiv (\neg p \vee p) \wedge (\neg p \vee q) \\ &\equiv (p \vee \neg p) \wedge (\neg p \vee q) \\ &\equiv \text{T} \wedge (\neg p \vee q)\end{aligned}$$

Stay on target:

We met our intermediate goal.

Don't forget the final goal!

We want to end up at  $(\neg p \vee q)$

If we apply the distribution rule,

We'd get a  $(\neg p \vee q)$

**Commutative**

*bring negation*

$$\equiv (\neg p \vee q)$$

Make the expression look exactly like the law (more on this later)

**Identity.**

Simplifies the part we want to disappear.

# Simplify $\top \wedge (\neg p \vee q)$ to $(\neg p \vee q)$

For every propositions  $p, q, r$  the following hold:

- **Identity**

- $p \wedge \top \equiv p$
- $p \vee \text{F} \equiv p$

- **Domination**

- $p \vee \top \equiv \top$
- $p \wedge \text{F} \equiv \text{F}$

- **Idempotent**

- $p \vee p \equiv p$
- $p \wedge p \equiv p$

- **Commutative**

- $p \vee q \equiv q \vee p$
- $p \wedge q \equiv q \wedge p$

- **Associative**

- $(p \vee q) \vee r \equiv p \vee (q \vee r)$
- $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

- **Distributive**

- $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
- $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

- **Absorption**

- $p \vee (p \wedge q) \equiv p$
- $p \wedge (p \vee q) \equiv p$

- **Negation**

- $p \vee \neg p \equiv \top$
- $p \wedge \neg p \equiv \text{F}$



# Our First Proof

$$(p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q) \equiv (p \wedge q) \vee [(\neg p \wedge q) \vee (\neg p \wedge \neg q)]$$

$$\equiv (p \wedge q) \vee [\neg p \wedge (q \vee \neg q)]$$

**Stay on target:**

We met our intermediate goal.

Don't forget the final goal!

We want to end up at  $(\neg p \vee q)$

$$\equiv (p \wedge q) \vee [\neg p \wedge T]$$

$$\equiv (p \wedge q) \vee [\neg p]$$

$$\equiv [\neg p] \vee (p \wedge q)$$

$$\equiv (\neg p \vee p) \wedge (\neg p \vee q)$$

If we apply the distribution rule,

We'd get a  $(\neg p \vee q)$

$$\equiv (p \vee \neg p) \wedge (\neg p \vee q)$$

$$\equiv T \wedge (\neg p \vee q)$$

$$\equiv (\neg p \vee q) \wedge T$$

$$\equiv (\neg p \vee q)$$

**Commutative** followed by **Identity**

Look exactly like the law, then apply it.

We're done!!!

# Commutativity

We had the expression  $(p \wedge q) \vee [\neg p]$

But before we applied the distributive law, we switched the order...why?

The law says  $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

not  $(q \wedge r) \vee p \equiv (q \vee p) \wedge (r \vee p)$

So **technically** we needed to commute first.

Eventually (in about 2 weeks) we'll skip this step. For now, we're doing two separate steps.

Remember this is the "training wheel" stage. The point is to be careful.

# More on Our First Proof

We now have an ironclad guarantee that

$$(p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q) \equiv (\neg p \vee q)$$

Hooray! But we could have just made a truth-table. Why a proof?

Here's one reason.

Proofs don't *just* give us an ironclad guarantee. They're also an explanation of *why* the claim is true.

The key insight to our simplification was “the last two pieces were the vacuous truth parts – the parts where *p* was false”

That's in there, *in the proof*.

# Our First Proof

$$\begin{aligned}(p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q) &\equiv (p \wedge q) \vee [(\neg p \wedge q) \vee (\neg p \wedge \neg q)] && \text{Associative} \\ &\equiv (p \wedge q) \vee [\neg p \wedge (q \vee \neg q)] && \text{Distributive} \\ &\equiv (p \wedge q) \vee [\neg p \wedge \text{T}] && \text{Negation} \\ &\equiv (p \wedge q) \vee [\neg p] && \text{Identity} \\ &\equiv [\neg p] \vee (p \wedge q) && \text{Commutative} \\ &\equiv (\neg p \vee p) \wedge (\neg p \vee q) && \text{Distributive} \\ &\equiv (p \vee \neg p) \wedge (\neg p \vee q) && \text{Commutative} \\ &\equiv \text{T} \wedge (\neg p \vee q) && \text{Negation} \\ &\equiv (\neg p \vee q) \wedge \text{T} && \text{Commutative} \\ &\equiv (\neg p \vee q) && \text{Identity}\end{aligned}$$

The last two terms are "vacuous truth" – they simplify to  $\neg p$

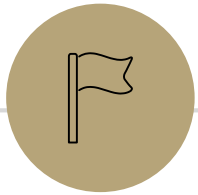
$p$  no longer matters in  $p \wedge q$  if  $\neg p$  automatically makes the expression true.

# More on Our First Proof

With practice (and quite a bit of squinting) you can see not just the ironclad guarantee, but also the reason why something is true.

That's not easy with a truth table.

Proofs can also communicate intuition about why a statement is true. We'll practice extracting intuition from proofs more this quarter.



# Modifying Implications

---

# Converse, Contrapositive

**Implication:**

**Contrapositive:**

If it's raining, then I have my umbrella.

$$p \rightarrow q$$

$\neg q \rightarrow \neg p$  If I don't have my umbrella, then it is not raining.

**Converse:**

**Inverse:**

If I have my umbrella, then it is raining.

$$q \rightarrow p$$

$\neg p \rightarrow \neg q$  If it is not raining, then I don't have my umbrella.

**How do these relate to each other?**

$p$	$q$	$p \rightarrow q$	$q \rightarrow p$	$\neg p$	$\neg q$	$\neg p \rightarrow \neg q$	$\neg q \rightarrow \neg p$
T	T						
T	F						
F	T						
F	F						

# Converse, Contrapositive

Implication:

$$p \rightarrow q$$

Converse:

$$q \rightarrow p$$

Contrapositive:

$$\neg q \rightarrow \neg p$$

Inverse:

$$\neg p \rightarrow \neg q$$

An **implication** and its **contrapositive**  
have the same truth value!

$p$	$q$	$p \rightarrow q$	$q \rightarrow p$	$\neg p$	$\neg q$	$\neg p \rightarrow \neg q$	$\neg q \rightarrow \neg p$
T	T	T	T	F	F	T	T
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	T	T	T



# Contrapositive

We showed  $p \rightarrow q \equiv \neg q \rightarrow \neg p$  with a truth table. Let's do a proof.

Try this one on your own. Remember

1. Know what you're trying to show.
2. Stay on target – take steps to get closer to your goal.

Hint: think about your tools.

There are lots of rules with AND/OR/NOT,  
but very few with implications...

[pollev.com/robbie](https://pollev.com/robbie)

Help me adjust my explanation!

# Contrapositive

$p \rightarrow q \equiv \neg p \vee q$	Law of Implication
$\equiv q \vee \neg p$	Commutativity
$\equiv \neg \neg q \vee \neg p$	Double Negation
$\equiv \neg q \rightarrow \neg p$	Law of Implication

All of our rules deal with ORs and ANDs, let's switch the implication to just use AND/NOT/OR.

And do the same with our target

It's ok to work from both ends. In fact it's a very common strategy!

Now how do we get the top to look like the bottom?

Just a few more rules and we're done!

# Work from both ends, but...

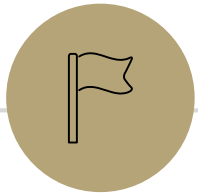
...make sure at the end, if you read from top-to-bottom, every step makes sense.

When proving an equivalence you must:

1. Start with the left side (or right side)
2. Modify what you had in the last step (using an equivalence)
3. Derive the right side (or left side if you started with the right)

You may **not** start with the equivalence you're trying to show, and simplify to something "obviously true."

More on why later in the quarter, but tl;dr for now is you can't use your goal as a starting assumption (it's what you're trying to show! If you knew it, no need to write a proof).



# Digital Logic



# Digital Circuits

## Computing With Logic

**T** corresponds to **1** or “high” voltage

**F** corresponds to **0** or “low” voltage

## Gates

Take inputs and produce outputs (functions)

Several kinds of gates

Correspond to propositional connectives (most of them)

# And Gate

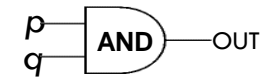
**AND Connective**

vs.

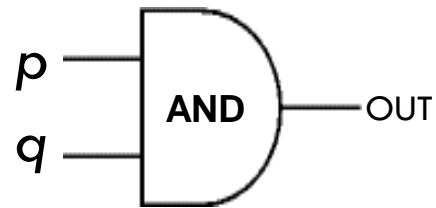
**AND Gate**

$p \wedge q$

$p$	$q$	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F



$p$	$q$	OUT
1	1	1
1	0	0
0	1	0
0	0	0



“block looks like D of AND”

# Or Gate

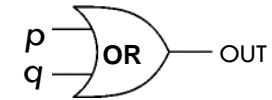
**OR Connective**

**vs.**

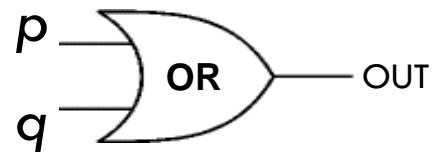
**OR Gate**

$p \vee q$

$p$	$q$	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F



$p$	$q$	OUT
1	1	1
1	0	1
0	1	1
0	0	0



“arrowhead block looks like V”

# Not Gates

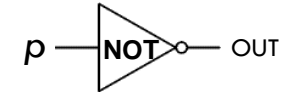
**NOT Connective**

vs.

**NOT Gate**

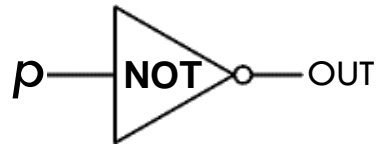
$\neg p$

$p$	$\neg p$
T	F
F	T



Also called  
*inverter*

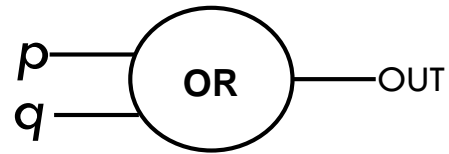
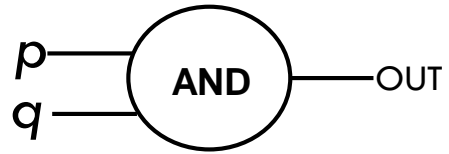
$p$	OUT
1	0
0	1



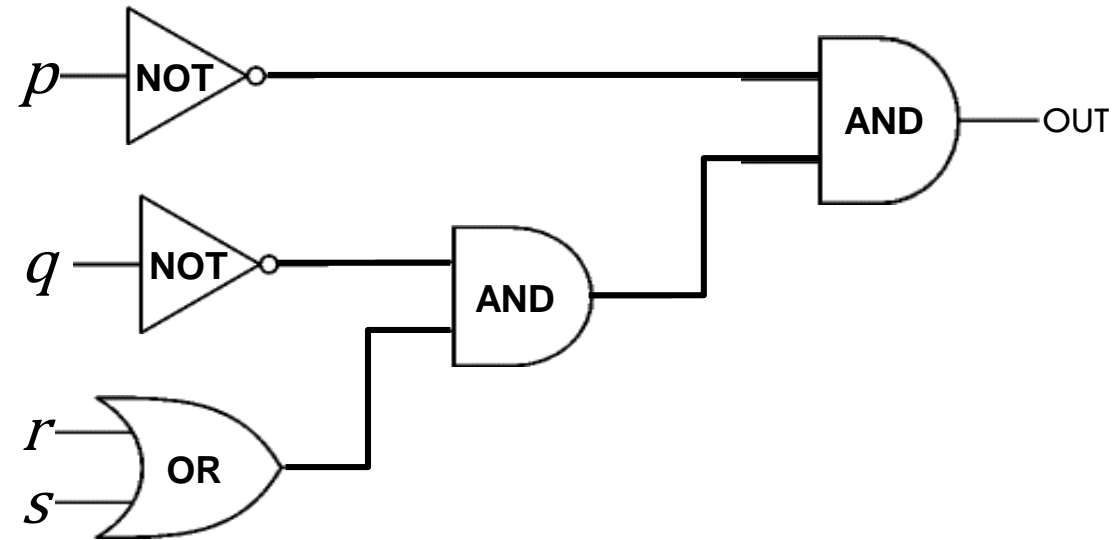


# Blobs are Okay!

You may write gates using blobs instead of shapes!

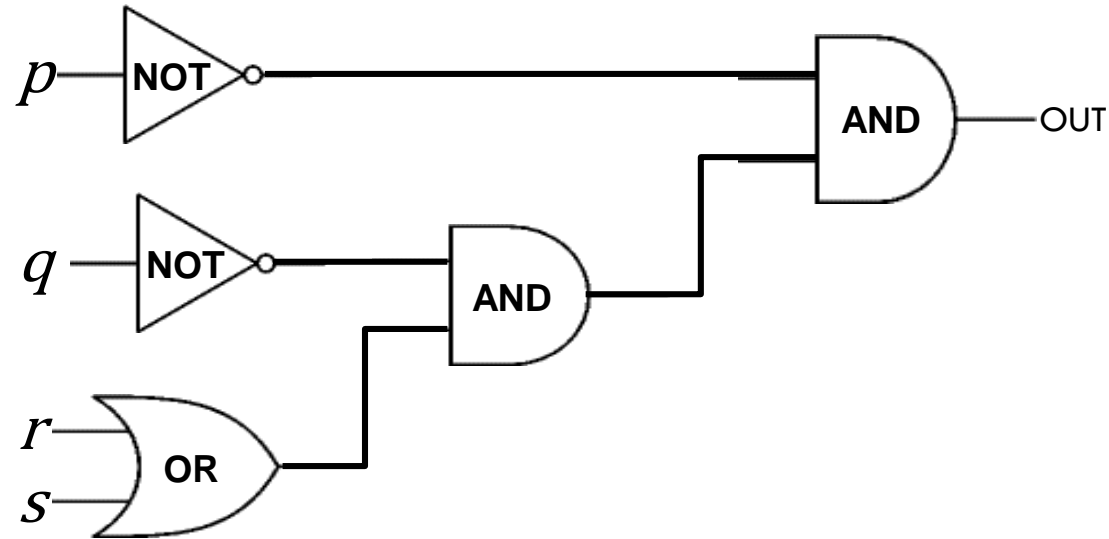


# Combinational Logic Circuits



**Values get sent along wires connecting gates**

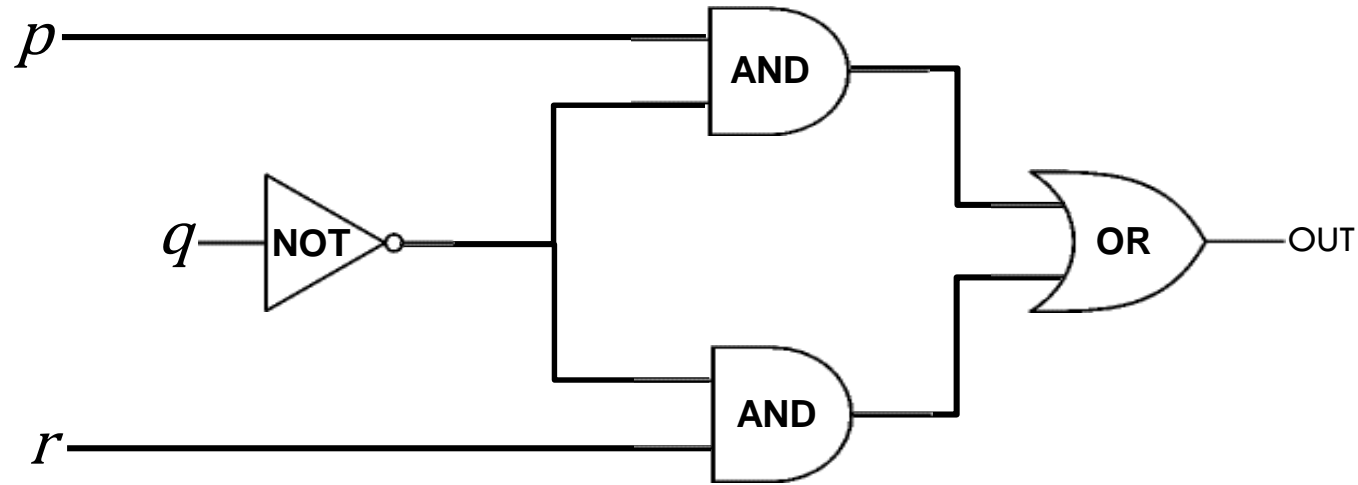
# Combinational Logic Circuits



**Values get sent along wires connecting gates**

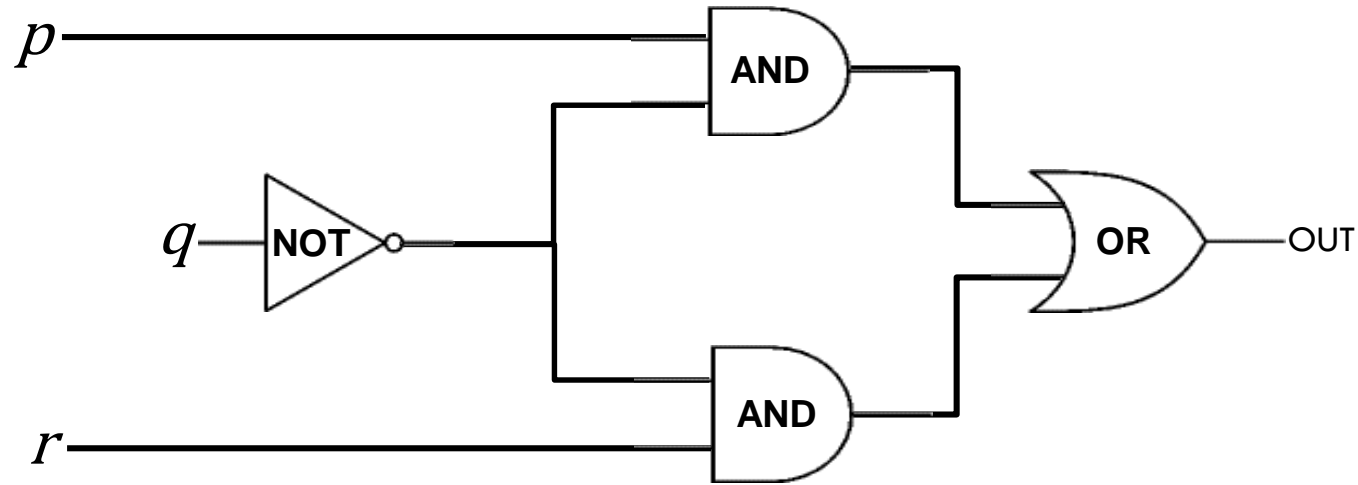
$$\neg p \wedge (\neg q \wedge (r \vee s))$$

# Combinational Logic Circuits



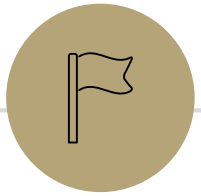
**Wires can send one value to multiple gates!**

# Combinational Logic Circuits



**Wires can send one value to multiple gates!**

$$(p \wedge \neg q) \vee (\neg q \wedge r)$$



## More Vocabulary

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# Vocabulary!

A proposition is a....

*Tautology* if it is always true.

*Contradiction* if it is always false.

*Contingency* if it can be both true and false.

$$p \vee \neg p$$

Tautology

If  $p$  is true,  $p \vee \neg p$  is true; if  $p$  is false,  $p \vee \neg p$  is true.

$$p \oplus p$$

Contradiction

If  $p$  is true,  $p \oplus p$  is false; if  $p$  is false,  $p \oplus p$  is false.

$$(p \rightarrow q) \wedge p$$

Contingency If  $p$  is true and  $q$  is true,  $(p \rightarrow q) \wedge p$  is true;  
If  $p$  is true and  $q$  is false,  $(p \rightarrow q) \wedge p$  is false.

# Another Proof

Let's prove that  $(p \wedge q) \rightarrow (q \vee p)$  is a tautology.

Alright, what are we trying to show?



# Another Proof

$$\begin{aligned}(p \wedge q) \rightarrow (q \vee p) &\equiv \neg(p \wedge q) \vee (q \vee p) \\ &\equiv (\neg p \vee \neg q) \vee (q \vee p) \\ &\equiv \neg p \vee (\neg q \vee (q \vee p)) \\ &\equiv \neg p \vee ((\neg q \vee q) \vee p) \\ &\equiv \neg p \vee ((q \vee \neg q) \vee p) \\ &\equiv \neg p \vee (T \vee p) \\ &\equiv \neg p \vee (p \vee T) \\ &\equiv \neg p \vee p \\ &\equiv p \vee \neg p \\ &\equiv T\end{aligned}$$

Proof-writing tip:

**Take a step back.**

Pause and carefully look at what you have. You might see where to go next...

Law of Implication

DeMorgan's Law

It's easier if everything is AND/OR/NOT  
Gets rid of some parentheses/just a gut feeling

Associative (twice)

Commutative Negation

Put  $q, \neg q$  next to each other.

Commutative Domination

Simplify out the  $q, \neg q$ .

Commutative Negation

Simplify out the  $p, \neg p$ .

We're done!

# Another Proof

$(p \wedge q) \rightarrow (q \vee p)$	$\equiv \neg(p \wedge q) \vee (q \vee p)$	Law of implication
	$\equiv (\neg p \vee \neg q) \vee (q \vee p)$	DeMorgan's Law
	$\equiv \neg p \vee (\neg q \vee (q \vee p))$	Associative
	$\equiv \neg p \vee ((\neg q \vee q) \vee p)$	Associative
	$\equiv \neg p \vee ((q \vee \neg q) \vee p)$	Commutative
	$\equiv \neg p \vee (T \vee p)$	Negation
	$\equiv \neg p \vee (p \vee T)$	Commutative
	$\equiv \neg p \vee p$	Domination
	$\equiv p \vee \neg p$	Commutative
	$\equiv T$	Negation

# Computing Equivalence

**Given two propositions, can we write an algorithm to determine if they are equivalent?**

**What is the runtime of our algorithm?**

# Computing Equivalence

Given two propositions, can we write an algorithm to determine if they are equivalent?

Yes! Generate the truth tables for both propositions and check if they are the same for every entry.

What is the runtime of our algorithm?

Every atomic proposition has two possibilities (T, F). If there are  $n$  atomic propositions, there are  $2^n$  rows in the truth table.