Homework 3: Predicate Logic

Due date: Wednesday January 25th at 10 PM
If you work with others (and you should!), remember to follow the collaboration policy outlined in the syllabus.
In general, you are graded on both the clarity and accuracy of your work. Your solution should be clear enough that someone in the class who had not seen the problem before would understand it.
We sometimes describe approximately how long our explanations are. These are intended to help you understand approximately how much detail we are expecting. You are allowed to have longer explanations, but explanations significantly longer than necessary may receive deductions.

Be sure to read the grading guidelines on the assignments page for more information on what we're looking for. **Be sure to read the guidelines on inference proofs on the assignments page. The requirements on some rules are looser now that we're nearing the end of the training wheels phase!**

1. Onion Debate

For those who are not interested in roller-skating, Robbie also owns a glorious Walla Walla sweet onion farm. Restrictions imposed by the Department of Agriculture require that only people who bring a shallot with them may enter this farm.

You let your domain of discourse be all people and plants in the farm.

The predicates Shallot, WallaWalla, Human are true if and only if the input is a shallot, a Walla Walla sweet onion, or a human respectively. It is a sunny day and Robbie is hosting a debate about whether Walla Walla sweet onions are better than shallots. The predicate $\texttt{SweetOnionLover}(x)$ means "x is a sweet onion lover" and similarly for $\texttt{ShallotLover}(x)$. Finally EnjoyingSun is true if and only if the input human/plant is enjoying the sun.

1.1. Round One[12 points]

Translate the following observations into English. Your translations should take advantage of "restricting the domain" to make more natural translations when possible, but you should not otherwise simplify the expression before translating.

These requirements mean that

- You must not use variable names in your English translation (e.g., don't say "for every x...")
- For every quantified variable where one or more of the predicates can be interpreted as a domain restriction, you must use at least one of them to make your translation more natural. So with a domain of discourse of all integers, $\forall x([\texttt{Even}(x) \land \texttt{Prime}(x)] \to \texttt{IsEqual}(x, 2)$ could be translated as "For every even integer, if it is prime it is equal to 2" or "Every prime and even integer is equal to 2" but could not be translate as "For every integer, if the integer is prime and even then it is equal to 2."
- If the sentence does not have domain restriction, you may use "everyone" or "someone" to refer to an arbitrary element of the domain.

(a) $\exists x\, (\texttt{WallaWalla}(x) \land \neg\, \texttt{SweetOnionLover}(x) \land \texttt{EnjoyingSun}(x))$

(b) $\forall x(\texttt{ShallotLover}(x) \lor \texttt{Shallot(x)} \to \neg\, \texttt{EnjoyingSun}(x)) \land \forall x(\texttt{SweetOnionLover}(x) \to \texttt{EnjoyingSun}(x))$

(c) $\neg\exists x(\texttt{Human}(x) \land \texttt{EnjoyingSun}(x) \land \texttt{ShallotLover}(x))$

1.2. Round Two [4 points]

You realize that the first sentence is false. State the negation of (a) in English. You should simplify the negation so that the English sentence is natural.

2. Become a Domain Expert [10 points]

For the following statements, translate them into predicate logic (specifying and defining any predicates you use). Then provide a domain of discourse where the statement is true and another domain of discourse where the statement is false.

Also include 1-2 sentences for each domain for why the statement has the truth value it does.

If you wish to make extra assumptions about the world (like "there is a cat named Garfield who is fat and unhappy") you may do so.

(a) Every x that speaks gets a treat.

(b) There is an x such that for all y, x is less than or equal to y.

3. Nested Quantifiers [15 points]

Fix your domain of discourse to be all mammals. Use the predicates $\texttt{Cat}(x), \texttt{Dog}(x), \texttt{Human}(x)$ to say x is a cat, dog, or human respectively. You can also use the predicates $\texttt{Happy}(x)$, $\texttt{Sad}(x)$, $\texttt{Fluffy}(x)$ to mean x is happy, sad, or fluffy respectively. Finally, the predicate $\texttt{IsPetOf}(x, y)$ means y is the pet of x (note the order – the pet goes second).

In this problem, an example of something you might give for a "scenario" might be "There is at least one cat who is happy and at least one cat who is not happy."

(a) Your friend tried to translate "Every cat is happy or fluffy" and got

$$\forall x(\texttt{cat}(x) \land [\texttt{happy}(x) \lor \texttt{fluffy}(x)]).$$

The translation is incorrect. Give a correct translation, and describe a scenario (i.e. facts about mammals) in which your translation and their translation evaluate to different truth values.

(b) Your friend tried to translate "There is a cat who is the pet of all humans" and got

$$\exists x \forall y([\texttt{cat}(x) \land \texttt{human}(y)] \to \texttt{pet}(y, x)).$$

The translation is incorrect. Give a correct translation, and describe a scenario (i.e. facts about widgets) in which your translation and their translation evaluate to different truth values.

(c) Translate the sentence "For every mammal x, there is a mammal y such that for every mammal z: y is a cat and x is the pet of z" into predicate logic.

4. Spoof [12 points]

This problem will be our first "spoof." A spoof is something that looks like a proof, but isn't (because there are one or more signifincat mistakes made along the way.

Theorem?: Given $\neg c, \neg b \to c, c \land \neg a \lor b$ can you conclude $\texttt{F}$? (Concluding $\texttt{F}$ would mean that the givens [or more formally, the conjunction of all the givens] is a contradiction).

“Spoof”:

1.	$\neg b \rightarrow c$	Given
2.	$\neg c \rightarrow \neg\neg b$	Contrapositive (1)
3.	$\neg c \rightarrow b$	Double Negation (2)
4.	$\neg c$	Given
5.	b	Modus Ponens (3,4)
6.	$c \wedge \neg a \vee b$	Given
7.	$c \wedge (\neg a \vee b)$	Associativity (6)
8.	$c \wedge (b \vee \neg a)$	Commutativity (7)
9.	$c \wedge (b \rightarrow a)$	Law of Implication (8)
10.	$c \wedge a$	Modus Ponens (5,9)
11.	$\neg c \wedge c \wedge a$	Intro $\wedge$ (4,10)
12.	$c \wedge \neg c \wedge a$	Commutativity (11)
13.	$\mathsf{F} \wedge a$	Negation (12)
14.	$a \wedge \mathsf{F}$	Commutativity (13)
15.	F	Domination (14)

There are (at least) three significant errors in the spoof above. List three significant errors. For each error, state the line number where the error first appears and why it’s incorrect (like “step 3, Modus Ponens needs $p \rightarrow q$ and p, we have $p \rightarrow q$ and q”). 1 sentence was enough for each of our explanations.

We tried to write a pristine spoof (i.e., we tried to make it so there are exactly three errors and nothing else wrong), but there may be (unintentional) insignificant errors along with the (intentional) significant ones. An insignificant error would be something like listing the wrong number in a prior step or forgetting to mention that commuativity was used along with another rule, or using the wrong name for a rule – something that could be fixed quickly.

A significant error is one that could lead to a false conclusion, or which requires inserting multiple steps to correct.

5. Inference Spoof [22 points]

Theorem: Given $\neg s \to (p \wedge q), s \to r$, and $(r \wedge p) \to q$, prove q.

"Spoof:"

1.	$\neg s \to (p \wedge q)$		Given
2.1.	$\neg s$	Assumption	
2.2.	$p \wedge q$	MP: 2.1 1	
2.3.	p	Elim of $\wedge$: 2.2	
2.4.	q	Elim of $\wedge$: 2.2	
3.	$\neg s \to q$		Direct Proof Rule
4.	$s \to r$		Given
5.	$(r \wedge p) \to q$		Given
6.1.	s	Assumption	
6.2.	r	MP: 6.1, 3	
6.3.	$r \wedge p$	Intro $\wedge$: 6.2, 2.3	
6.4.	q	MP: 6.3, 5	
7.	$s \to q$		Direct Proof Rule
8.	$(s \to q) \wedge (\neg s \to q)$		Intro $\wedge$: 7, 3
9.	$(\neg s \vee q) \wedge (\neg\neg s \vee q)$		Law of Implication
10.	$(\neg s \vee q) \wedge (s \vee q)$		Double Negation
11.	$((\neg s \vee q) \wedge s) \vee ((\neg s \vee q) \wedge q)$		Distributivity
12.	$((\neg s \vee q) \wedge s) \vee (q \wedge (\neg s \vee q))$		Commutativity
13.	$((\neg s \vee q) \wedge s) \vee (q \wedge (q \vee \neg s))$		Commutativity
14.	$((\neg s \vee q) \wedge s) \vee q$		Absorption
15.	$q \vee ((\neg s \vee q) \wedge s)$		Commutativity
16.	$(q \vee (\neg s \vee q)) \wedge (q \vee s)$		Distributativity
17.	$(q \vee (q \vee \neg s)) \wedge (q \vee s)$		Commutativity
18.	$q \wedge (q \vee s)$		Absorption
19.	q		Absorption

(a) There are (at least) two significant errors in this proof. Indicate which lines contain the errors and, for each one, explain why that line is incorrect. We only needed about 1 sentence to explain each incorrect line. [8 points]

(b) Is the conclusion of the "spoof" correct (that is, is the "Theorem" true)? If it is incorrect, describe propositions p, q, r, s such that the givens are true, but the claim is false. If the conclusion is correct, briefly explain how to correct any errors in lines 1–8 (you'll explain errors in 8–19 in part c). [4 points]

(c) Give a correct proof of what is claimed in lines 8–19, i.e., that from $(s \to q) \wedge (\neg s \to q)$, we can infer that q is true. [10 points]

6. Logic: Not just for computer scientists [12 points]

Background

Our main goal in 311 is to prepare you to be a better computer scientist, but some of the lessons of the course are useful in "everyday life." One of the most common errors in reasoning in the "real world" is confusing an implication for its converse. That is, thinking that $p \to q$ and $q \to p$ can always be interchanged. This error can appear in a few ways.

One way is a mistake called "affirming the consequent." The error is from givens $p \to q$ and q to conclude p. That conclusion is an error – from q and $q \to p$ (the converse of $p \to q$), one can conclude p. But not from q and $p \to q$.

This mistake can also appear in much longer strings of logical reasoning. For example, it is tempting to combine "If it is raining, then we won't play softball."; "If we don't play softball, then we'll get ice cream"; "we got ice cream" into the conclusion "it is raining.", but it might not be raining! (Maybe we get ice cream if we don't play softball OR if we play softball and win).

A common reason for this error is assuming that one cause is the only possible cause. Of course, this doesn't mean the conclusion is necessarily wrong! It may be that p is in fact true! But we can't guarantee it from just the reasoning given.

6.1. In Real Life [6 points]

Find an example of someone making an error involving mixing up an implication and its converse. This could be in real life (like a politician or opinion piece), in culture (like another TV clip or an advertisement), or just something your friend said. If you can't think of one, you may use one of the examples in the last section.

(a) Write down a quote of the person making the logical error.

(b) If you can link us to a source, do so here (don't worry about formatting). If you can't (because there's no record of the statement), just say that.

(c) From the quote you have in (a), define propositions p, q and translate it to propositional logic. What givens are they asserting and what is their conclusion (i.e. what do they intend to argue for)? Double check that they really are making an error.

(d) Suggest replacing one (or more) of the implications with their converse in the propositional logic you have in (c). With the change, can one reach their desired conclusion without making any logical error? (1-3 sentences)

(e) Do you think the converse(s) you inserted are true? Or at least "often true"?[1] Explain in 1-3 sentences.

Some options

You're encouraged to keep your eyes out for this error in real life! Or to think about places you might have seen it. If you cannot find one, you might choose one of these options instead:

- This clip from The Simpsons.
- This debate comment from then-presidential-candidate Mike Huckabee.
- This clip from the childrens' show "If you give a mouse a cookie."

[1] For example, it might not be true that "If I have my umbrella, then it is raining" (since I also bring my umbrella when it snows), but snow is so rare that the implication would 'often' be true.

6.2. At UW [6 points]

Take (and attach) a photo of some sign (preferably at UW, but we will accept any signs you see in your day-to-day life), break the contents of the sign into atomic propositions and provide the logical translation of this sign. Please also mention where you found this sign (so perhaps if any of your TA's are bored, we can do a logical sign trek :D). Example:

.

We can represent this with the atomic propositions :
p : One is an unattended child.
q : One will be given an espresso.
r : One will be given a free puppy.
$p \to q \wedge r$

(a) Attach a photo of some sign / iconography you have found.

(b) If you feel comfortable, mention where you found this sign.

(c) Define atomic propositions.

(d) Translate the contents of the sign into logic using the atomic propositions defined above.

7. Feedback [1 point]

Please keep track of how much time you spend on this homework and answer the following questions. This can help us calibrate future assignments and future iterations of the course, and can help you identify which areas are most challenging for you.

- How many hours did you spend working on this assignment (excluding any extra credit questions, if applicable)? Report your estimate to the nearest hour.
- Which problem did you spend the most time on?
- Any other feedback for us?