CSE 311: Foundations of Computing I

Section 7: Contradiction, Strong Induction, Structural Induction

1. How Contradictory!

Recall that we defined the **rational numbers** as the set $\mathbb{Q} = \{\frac{a}{b} : a, b \in \mathbb{Z}, b \neq 0\}$. For example, $\frac{5}{8}, -12$ are rational.

The **irrational numbers** are all real numbers that are not rational. For example, $\sqrt{2}, \pi$ are irrational.

Prove by contradiction that the sum of a rational number with an irrational number is always irrational.

2. Walk the Dogs

Suppose that a dog walker takes care of $n \geq 12$ dogs. The dog walker is very superstitious, and will walk dogs in groups of 3 or 7 at a time (every dog gets walked exactly once). Prove that the dog walker can always split the n dogs into groups of 3 dogs or 7 dogs.

3. Recursively Defined Sets

For each of the following, write a recursive definition of the sets satisfying the following properties. Briefly justify that your solution is correct.

(a) Binary strings of odd length.

(b) Binary strings not containing 10.

(c) Binary strings not containing 10 as a substring and having at least as many 1s as 0s.

4. Seeing Double

Recall the following recursive definition of the set of strings Σ^* from lecture:
Basis Step: $\varepsilon \in \Sigma^*$
Recursive Step: If $w \in \Sigma^*$ and $a \in \Sigma$, then $wa \in \Sigma^*$.

Now consider the following recursive definitions of the functions len and double:
$\text{len}(\varepsilon) = 0$
$\text{len}(wa) = \text{len}(w) + 1$

$\text{double}(\varepsilon) = \varepsilon$
$\text{double}(wa) = (\text{double}(w))aa$

Prove that for every string $s \in \Sigma^*$, $\text{len}(\text{double}(s)) = 2 \cdot \text{len}(s)$

5. Reversing a Binary Tree

Recall the following recursive definition of the set of Trees from lecture:
Basis Step: null $\in$ Tree
Recursive Step: If $L, R \in$ Tree and $a \in \mathbb{Z}$, then $(L, a, R) \in$ Tree.

Now consider the following recursive definitions of the functions sum and reverse:
$\text{sum}(\text{null}) = 0$
$\text{sum}((L, a, R)) = a + \text{sum}(L) + \text{sum}(R)$

$\text{reverse}(\text{null}) = \text{null}$
$\text{reverse}((L, a, R)) = (\text{reverse}(R), a, \text{reverse}(L))$

Prove that for every Tree $T \in$ Tree that $\text{sum}(\text{reverse}(T)) = \text{sum}(T)$