CSE 311: Foundations of Computing I

Set Theory

Common Sets

- $\mathbb{N} = \{0, 1, 2, \ldots\}$ is the set of *Natural Numbers*.
- $\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$ is the set of *Integers*.
- $\mathbb{Q} = \{p/q \, : \, p, q \in \mathbb{Z} \wedge q \neq 0\}$ is the set of *Rational Numbers*.
- $\mathbb{R}$ is the set of *Real Numbers*.

Set Notation

- "$x \in S$" means x is an element of the set S.
- "$x \notin S$" means $\neg(x \in S)$, i.e. x is not an element of the set S.
- $S = \{x \, : \, \mathsf{P}(x)\}$ is *set builder notation*, and means that S is the set that contains all objects x that make $\mathsf{P}(x)$ true.

Set Operations

Let A, B be sets. We can define new sets from A and B:

- $A \cup B$ is the *union* of A and B: $A \cup B \; = \{x \, : \, x \in A \vee x \in B\}$
- $A \cap B$ is the *intersection* of A and B: $A \cap B \; = \{x \, : \, x \in A \wedge x \in B\}$
- $A \setminus B$ is the *difference* of A and B: $A \setminus B \; = \{x \, : \, x \in A \wedge x \notin B\}$
- $\overline{A}$ is the *complement* of A with respect to "universe" $\mathcal{U}$: $\overline{A} \; = \{x \in \mathcal{U} \; : \; x \notin A\}$
- $A \times B$ is the *Cartesian product* of A and B: $A \times B \; = \{(a, b) \; : \; a \in A, \; b \in B\}$
- $\mathcal{P}(A)$ is the *Power Set* of A, which is the set of all subsets of A: $\mathcal{P}(A) \; = \{B \, : \, B \subseteq A\}$

Set Comparison

Let A, B be sets. We can define new predicates that compare A and B:

- A *equals* B when they have the same elements: $A = B \; = \forall x \, (x \in A \leftrightarrow x \in B)$
- A is a *subset* of B when B contains all of A's elements: $A \subseteq B \; = \forall x \, (x \in A \rightarrow x \in B)$
- $A = B$ iff $A \subseteq B$ and $B \subseteq A$