

Induction Practice

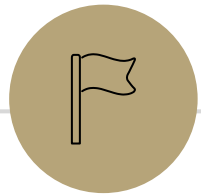
CSE 311: Foundations of
Computing I
Lecture 16

Announcements

- HW5 due tonight at 11:59 pm.

Turn it in with no late days to receive feedback by tomorrow. If you submit HW5 now, you **cannot** submit it with late days again later.

- Midterm is on Friday in class.
 - Recording of review session has been posted.
 - Section tomorrow will be Midterm Review. Problems & solutions are posted already for extra practice.
- No new HW released this week.



Find the Bug

Find the Bug

Claim: For every odd integer n , $n^2 \equiv_4 1$.

Proof: Let n be an arbitrary odd integer. Then by definition of odd, $n = 2k + 1$ for some integer k . Then consider $n^2 \equiv_4 1$. Plugging in $n = 2k + 1$ for n^2 :

$$\begin{aligned}n^2 &\equiv_4 1 \\(2k + 1)^2 &\equiv_4 1 \\4k^2 + 4k + 1 &\equiv_4 1\end{aligned}$$

Then by definition of congruence, $4 \mid 4k^2 + 4k + 1 - 1$, so $4 \mid 4k^2 + 4k$. Since this is true, the claim holds.

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Backwards Reasoning:
Assumes the statement
we're trying to prove is
true.

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Fixed Proof

Claim: For every odd integer n , $n^2 \equiv_4 1$.

Proof: Let n be an arbitrary odd integer. Then by definition of odd, $n = 2k + 1$ for some integer k . Then consider n^2 :

$$n^2 = (2k + 1)^2 = 4k^2 + 4k + 1$$

$$n^2 - 1 = 4k^2 + 4k$$

$$n^2 - 1 = 4(k^2 + k)$$

Since k is an integer, $k^2 + k$ is an integer. So by definition of divides, $4 \mid n^2 - 1$. So by definition of congruence, $n^2 \equiv_4 1$. Since n was arbitrary, the claim holds.

Backwards Reasoning

Backwards reasoning is the incorrect proof technique of *assuming* the goal is true, and then deriving some other true statement.

This reasoning can be used to incorrectly prove false statements.

Claim: For all integers x , if $x^2 = 25$, then $x = 5$.

Backwards Proof: Let x be an arbitrary integer. Suppose $x^2 = 25$.

Plugging in $x = 5$, we have $5^2 = 25$. Since this is true, the claim holds.

False! What if $x = -5$?

Find the 5 Bugs

Claim: For all integers $n \geq 1$, $1 + \dots + n = \frac{n(n+1)}{2}$.

Proof: Let $P(n)$ be " $1 + \dots + n = \frac{n(n+1)}{2}$ for all integers $n \geq 1$ ". We prove by induction.

Base Case: Plugging in $n = 1$, we have $1 = \frac{1(1+1)}{2}$. So $1 = \frac{2}{2}$. So $1 = 1$. Since this is true, the base case holds.

IH: Suppose $1 + \dots + k = \frac{k(k+1)}{2}$ for an integer k .

IS: We aim to show $P(k + 1)$. Observe that:

$$1 + \dots + (k + 1) = 1 + \dots + k + (k + 1) = \frac{k(k+1)}{2} + (k + 1) = \frac{k(k+1) + 2(k+1)}{2} = \frac{(k+1)(k+2)}{2}$$

So $P(k + 1)$ holds.

Conclusion: Thus $P(n)$ holds for all integers $n \geq 1$ by induction.

Find the 5 Bugs

Definition of $P(n)$:
Shouldn't include "for all n "
inside the definition of P .

Claim: For all integers $n \geq 1$, $1 + \dots + n = \frac{n(n+1)}{2}$.

Proof: Let $P(n)$ be " $1 + \dots + n = \frac{n(n+1)}{2}$ for all integers $n \geq 1$ ". We prove by induction.

Base Case: Plugging in $n = 1$, we have $1 = \frac{1(1+1)}{2}$. So $1 = \frac{2}{2}$. So $1 = 1$. Since this is true, the base case holds.

Backwards Reasoning:
Shouldn't assume base case holds when proving it

IH: Suppose $1 + \dots + k = \frac{k(k+1)}{2}$ for an integer k .

Should be $k \geq 1$

IS: We aim to show $P(k + 1)$. Observe that:

k should be arbitrary

$$1 + \dots + (k + 1) = 1 + \dots + k + (k + 1) = \frac{k(k+1)}{2} + (k + 1) = \frac{k(k+1) + 2(k+1)}{2} = \frac{(k+1)(k+2)}{2}$$

So $P(k + 1)$ holds.

Didn't cite where we used the IH

Conclusion: Thus $P(n)$ holds for all integers $n \geq 1$ by induction.

Avoiding Backwards Reasoning in the Base Case

Incorrect Technique: Backwards reasoning

Plugging in $n = 1$, we have $1 = \frac{1(1+1)}{2}$. So $1 = \frac{2}{2}$. So $1 = 1$. Since this is true, the base case holds.

Valid Technique 1: Separating LHS and RHS

The LHS evaluates to 1. The RHS evaluates to $\frac{1(1+1)}{2} = \frac{2}{2} = 1$. Since $1 = 1$, the base case holds.

Valid Technique 2: Start from Left, convert to the Right

Observe that $1 = \frac{2}{2} = \frac{1 \cdot 2}{2} = \frac{1(1+1)}{2}$. So the base case holds.